MULTIPLICATION FACTS AND THE INTERMEDIATE ALGEBRA STUDENT

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by

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DEDICATION

This endeavor is dedicated to my wife, my children, my parents, my friends, and my God. Great feats are more joyful, more meaningful, and more celebratory with a loving support system. To me, this was a great feat. The love and encouragement I received from all of you over these four years has been overwhelming and has touched me deeply. Thank you, from start to finish!

ABSTRACT

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The present dissertation was written in journal-ready format to understand multiplication fact automaticity (MFA) within the domain of Intermediate Algebra and the extent to which MFA is related to student completion and/or success. The studies were conducted at a small, public university in the southwestern United States. Implications included the consideration of MFA (at or above the sample median of 94%) in placement, instructional, and curricular practices to better equip developmental mathematics students for success in Intermediate Algebra.

In the first study, the author examined whether students with high MFA were more successful in Intermediate Algebra than those with low MFA. Even though scientific calculators were permitted throughout the course and on all tests, the average unit test scores and end-of-course grades were statistically significantly higher for students who scored at or above the sample's median score (high) on an MFA test than students below the median (low). The sample consisted of 448 students enrolled in Intermediate Algebra. Placement scores, attendance, time-lapse between institutional enrollment and first mathematics course, and withdrawal decisions were not statistically significantly correlated with MFA status.

The purpose of the second study was to determine whether differences occurred between medians of MFA scores and student competencies on five specific types of problems from Intermediate Algebra assessments: (1) linear equation with fractions, (2) system of linear equations, (3) factor by grouping, (4) simplify a rational expression, and

(5) simplify a radical expression. Purposive sampling was used and resulted in the participation of 365 Intermediate Algebra students. Statistically significant differences existed on MFA median scores between groups for problems 3, 4, and 5. In contrast, no statistically significant differences existed on MFA scores for problems 1 and 2.

The purpose of the third study was to explore, through personal interviews, the lived experiences of students who withdrew from Intermediate Algebra. Purposive sampling techniques were used to invite eight students to participate in this phenomenological study. Interviews were transcribed verbatim resulting in 123 significant statements. Findings revealed six emerging themes: student goals, false course-expectations, the decision to withdraw, mathematics experiences, strategies for success, and mathematics self-efficacy.

KEY WORDS: Multiplication, Automaticity, Fluency, Multiplication facts, Developmental math, Attrition, Withdraw, Self-efficacy.

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I would like to acknowledge the tremendous faculty and staff at Sam Houston State University. You have truly developed this teacher into a scholar. I appreciate your commitment to rigor, mindset, and timeliness. I have felt completely supported and equipped throughout this process as a result of your hard work and persistence.

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PREFACE

After countless years of watching developmental mathematics students struggle, fail, and/or quit, I was motivated to understand what, if anything, could be done without losing academic rigor. Time and again, I noticed a different behavior from students who had memorized their multiplication facts compared to those who had not. This study gave me a venue to put my hypotheses through the scientific approach of data collection, participant selection, and statistical analyses. Comparing my results to existing literature has enabled me to qualify my findings and contribute to the gap of research about the role multiplication-fact automaticity plays in developmental mathematics success.

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CHAPTER I

INTRODUCTION

This dissertation follows the style and format of *Research in the Schools (RITS)*.

Developmental mathematics has been scrutinized in recent years for its high failure and attrition rates (Bailey, 2009; Bonham & Boylan, 2012; Cafarella, 2014). Institutions across the nation have attempted to redesign their programs and courses to shorten the amount of time needed to place into college-level mathematics courses; streamline curricula with less redundancy; offer supplemental and co-requisite courses; and address specific needs of developmental students, to include academic and affective characteristics. These efforts are fueled by a desire to increase student success in developmental mathematics. This research explores the possibility that a lack of multiplication fact knowledge may contribute to student behaviors that ultimately result in course failure or withdrawal.

While multiplication of integers is a basic skill embedded within typical Intermediate Algebra course objectives, there is little research identifying the benefits of high multiplication fact automaticity (MFA) over the use of multiplication fact retrieval tools, such as calculators, counting techniques, and/or multiplication charts. In this study, high automaticity is defined as having memorized the information well enough to have quick, retrievable access with little cognitive effort, and is measured using a timed, multiplication-fact assessment without access to retrieval tools. The objective of this research was to understand MFA as a topic within the domain of Intermediate Algebra to explore the extent, if any, to which automaticity is related to student completion and/or success. Three separate studies were conducted: (a) examining differences of MFA test results among eight comparison variables (the number of semesters lapsed between initial college enrollment and enrollment in a mathematics course, mathematics placement scores, first three-semester test scores, end-of-course grade, and attendance); (b) comparing medians between MFA scores and five Intermediate Algebra unit test problems; and (c) capturing the human experiences of eight students who withdrew from an Intermediate Algebra course. The researcher hoped to contribute to the existing literature by providing statistically sound, peer-reviewed evidence relevant to the argument of intentional MFA instruction and/or assessment protocols to improve student success, retention, and the overall experience in developmental mathematics.

As a basis for the overarching motif of MFA, the literature was reviewed and grouped into themes of interest, beginning with the settings where MFA was formally included. The importance of MFA, as it relates to acquisition of future mathematical concepts, anxiety, and neuroscience, were characterized in the second theme. Lastly, the information was synthesized through the lens of developmental mathematics programs and student outcomes (Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014; Wallace & Gurganus, 2005).

Settings

Primary education systems in top performing countries, U.S. primary schools, and learning disability programs are three settings where MFA frequently is included in the instruction. Although some post-secondary institutions may offer MFA formally or informally, research of the literature did not indicate any empirically based findings of such, nor present a measured correlation of the effect MFA may have on student retention, success, and/or completion. In China, the cultural value of studying hard includes MFA in the lower primary grades. In fact, Shanghai, China, who ranked number 1 overall on the 2012 Programme for International Student Assessment (PISA) results list, performed the equivalent of 2 school years ahead of Massachusetts, one of the strongest U.S. states, in mathematics (Organisation for Economic Cooperation and Development [OECD], 2012). Hong-Kong and Japan, ranking 3rd and 7th respectively in PISA's assessment (OECD, 2012), master MFA by the end of the 4th grade (Wurman & Wilson, 2012). Notwithstanding, researchers have discovered a strong sentiment among Chinese teachers that studying hard is more paramount to mathematics success than innate intelligence (Achieve, Inc., 2013).

U.S. schools typically cover MFA by the end of the 5th grade. However, prior to the implementation of Common Core Standards (Center for Public Education, 2013), few states realized the connection between elementary school arithmetic and college-level mathematics, evidenced by the fact that less than 15 states explicitly required MFA. Additionally, the widespread use of calculators sent a message that MFA was unnecessary (Wurman & Wilson, 2012) and may be contributing to diminished arithmetic proficiency (Cafarella, 2014). According to Wurman and Wilson (2012), "this is the equivalent of making career decisions for 4th grade students" (p. 47). They go on to say that, "Arithmetic is the foundation. Arithmetic has to be the priority and it has to be done right" (Wurman & Wilson, 2012, p. 47).

Based on the Common Core Standards, which was endorsed by former United States President Barack Obama, government officials, the U.S. Chamber of Commerce, and prominent education groups, 46 states have elected to implement new benchmarks to ensure college- and career-ready high school graduates (Center for Public Education, 2013). Among these benchmarks, "is the ability to reason and communicate mathematically" (Center for Public Education, 2013, p. 5). Whether MFA will become a requirement in support of this benchmark remains to be seen. The Common Core

Standards are clear about goals and results but leave curriculum decisions to individual states (Center for Public Education, 2013).

Learning disability programs are more deliberate in their intention of teaching MFA. The amount of research available on this topic is abundant and consistent with the belief that MFA is pivotal in the eventual development of higher-order mathematics skills and attainment of complex, problem-solving strategies. One such study conducted by De Visscher and Noel (2014) highlighted dyscalculia, which is a mathematics-specific diagnosis when factors such as intelligence, education, and sensory deficits are eliminated. Specifically, they made some discoveries as to the effect hypersensitivity-to-interference has on automaticity of basic mathematics facts. According to the National Association of Developmental Education (NADE), developmental education “focuses on the intellectual, social, and emotional growth and development of all students”. However, knowledge of instructional MFA methods specific to those with documented disabilities is meaningful. The most promising information about acquisition of MFA is that it can be remediated with mainstreamed as well as learning disabled students (Rouse, 2014).

Importance

The OECD (2012) summarized that U.S. students need to improve higher-order mathematical performance while maintaining focus on basic skills. To make viable contributions within their own communities, students must recognize knowledge, skills, and experiences in themselves, and be equipped to approach problems with curiosity and a personal, as well as academic investment (Pappano, 2014). Although MFA is recognized as an important objective by several national teaching organizations, including the National Council of Teachers of Mathematics and the National Mathematics Advisory Panel (Nelson, Burns, Kanive, & Ysseldyke, 2013; Poncy, McCallum, & Schmitt, 2010; U. S. Department of Education, 2008), a preferred method of instructing MFA has yet to be decided at any instructional level. A common ground gaining in popularity is a cognitive approach that combines Skinner's behaviorism with Piaget's constructivism (Alexander & Mayer, 2011). Interestingly, this blend of learning theories embodies the concept of automaticity. Further discussion about teaching methodology is beyond the scope of this review but would be a worthwhile endeavor if MFA is added to developmental math curricula. The benefits of acquiring such knowledge include gaining fluency in more complex mathematics concepts while experiencing less frustration and anxiety (Cafarella, 2014; Cates & Rhymer, 2003; Nelson et al., 2013; Poncy et al., 2010).

Rouse's research (2014) of a specific type of MFA instructional venue was in response to Michigan's recent adoption of The Common Core Standards. One of the reasons the citizens of Michigan chose to implement more rigorous standards was to better prepare students for college and careers. Despite the ambiguity in the Standards, Michigan's leadership interpreted them as to require MFA by the end of the 4th grade (Rouse, 2014), touting the desire for students to be able to solve complex problems without the barrier of basic fact interference. Their assessment of the issue was supported by the unified voice of many middle and high school teachers expressing grave concern over their students' inability to solve complex, higher-order mathematics problems due to lack of MFA (Rouse, 2014).

Carr and Alexeev (2011) conducted a longitudinal study of 206 second-grade students over a three-year period. The students represented 38 classrooms from seven schools in three counties in the state of Georgia. Ethnicity, gender and socioeconomic status were considered, and the sample was a relatively diverse cross-section of the American population. The point of the study was to determine whether gender, as well as arithmetic fact fluency or accuracy contributed to growth in mathematical, problem-solving strategies. Consistent with previous studies, Carr and Alexeev (2011) discovered that fluency and accuracy are important factors in the development of strategies used to solve complex, mathematical problems. In support of a national focus to improve accuracy and fluency in elementary schools, Carr and Alexeev (2011) reported, "…early fluency on basic arithmetic facts has long-term consequences for the development of more advanced strategies and for later mathematics competency" (p. 627). Alternative strategies, such as finger-counting or calculator use are not equivalent and could render the student ill prepared into adulthood (Nelson et al., 2013).

Campbell and Alberts (2009) conducted an analysis of variance on 44 adults (M_{age} = 20.2) and 24 adults (M_{age} = 25) on the topic of retrieval efficiency in addition/subtraction facts and multiplication/division facts, respectively. They concluded that in arithmetic, the preferred strategy is retrieval as opposed to other procedural strategies. Retrieval is superior due to speed, accuracy, and a smaller demand on the working memory (Campbell & Alberts, 2009).

Students who have acquired MFA might also experience less anxiety as it relates to math skills (Cates & Rhymer, 2003). In an interview conducted by Boylan (2011), Paul Nolting stated that anxiety and working memory are negatively correlated. In addition, anxiety retards the movement of information through long-term memory and abstract reasoning (Boylan, 2011). Furthermore, less fluency was found in students with higher levels of anxiety (Cates & Rhymer, 2003). One of the best methods to alleviate mathematics anxiety is to address self-efficacy, and one of the most powerful predictors of self-efficacy is mastery experiences that are gained through previous successes and failures (Bandura, 1997). In an attempt to increase the number of successes in mathematics, educators are encouraged to identify gaps in mathematics fluency and provide interventions to build fluencies as a possible means to reduce anxiety (Cates & Rhymer, 2003).

In its infancy, the field of neuroscience has found MFA to be an optimal brain mapping study. Researchers have discovered that arithmetic facts retrieved from long-term memory stimulate a separate area in the brain than arithmetic computations (Alexander & Mayer, 2011). A direct correlation has yet to be researched but the similarities between the discoveries in neuroscience and the concept that automaticity allows freedom to the working part of the brain to perform more complex operations is intriguing.

Developmental Mathematics

There were two developmental mathematics courses offered at the institution participating in this study: Pre-Algebra and Intermediate Algebra. The developmental spectrum started at arithmetic and covered the high school algebra sequence containing both pre-MFA and post-MFA topics. A passing grade of 75% on a similar MFA skills test was a requirement to receive a grade in the Pre-Algebra course, but MFA was not specifically taught in either developmental mathematics courses, and less than 3% of the sample took the Pre-Algebra course. In his dissertation, Djemil (2010) shared the results of research conducted on the effect memorizing multiplication tables had on high school performance in mathematics. The findings indicated that students who did not memorize multiplication facts scored lower on the comprehensive assessment than students who did, regardless of calculator availability and usage. In light of the current state of developmental mathematics students and programs, questions arise about whether findings similar to Djemil's among post-secondary students could change the sentiment about MFA inclusion in Pre-Algebra or as a pre-requisite module to Intermediate Algebra. According to Wallace and Gurganus (2005), "students who master their multiplication facts have a more positive attitude about their mathematics abilities and further mathematics experiences" (p. 33). Given MFA is as effective in the postsecondary setting for Djemil's (2010) sample, implementing it into the formal curriculum could improve retention, increase degrees earned, contribute to our society, and render the US more internationally competitive in the areas of math, science, and technology (OECD, 2012).

Limitations

The study was limited to a single, predominantly White, public institution. Additionally, participation was based on student willingness, availability, and consent. Due to these limitations, generalization of the findings is precluded; however, findings may show consistency with trends in developmental mathematics programs. A more expanded discussion of limitations was addressed in each manuscript.

Organization of Document

This dissertation was written in a journal-ready format, meaning each of Chapters 2, 3, and 4 represent separate research manuscripts ready for publication. The over-arching theme of the document is MFA, which provides a thread of continuity throughout the document. Chapter 1 contains a general introduction, brief literature review of MFA, and limitations. Chapter 5 discusses an integrated conclusion of the three individual studies, as well as recommendations for future research. The three middle chapters describe all aspects of their respective research topics, which include problem statements; theoretical frameworks; purposes; educational significance; research questions and designs; participant selection; instrumentation; procedures; and data analyses.

Although each study is unique, the participants were selected from the same sampling frame, and some instruments were used in more than one study. Chapters 2 and 3 are both quantitative studies, employing independent *t*, chi-squared and Mann-Whitney U tests. Chapter 4 is a qualitative, phenomenological study using general coding practices to interpret transcribed interviews of participants. All three manuscripts provide a self-contained list of references, while an exhaustive list for the entire document is located at the end.

CHAPTER II

A QUANTITATIVE INVESTIGATION OF MULTIPLICATION FACT AUTOMATICITY

This dissertation follows the style and format of *Research in the Schools (RITS)*.

Abstract

The intent of this study was to illuminate if Intermediate Algebra students with high multiplication fact automaticity were more successful in their developmental mathematics course than those with low multiplication fact automaticity. Even though scientific calculators were permitted throughout the course and on all tests, average unit test scores and end-of-course grades were statistically significantly higher for students who scored at or above the sample's median score (high) on a multiplication fact automaticity test than students below the median (low). Findings suggest student success rates could increase in Intermediate Algebra by requiring multiplication fact automaticity as a prerequisite. Placement scores, attendance; time-lapse between institutional enrollment and first mathematics course and withdrawal decisions were not related with multiplication fact automaticity status.

KEY WORDS: Multiplication, Automaticity, Fluency, Multiplication facts.

Arithmetic skills, to include multiplication-fact knowledge, have been linked to student success and psychological affect in a variety of ways (Wallace & Gurganus, 2005; Wurman & Wilson, 2012). While these findings have been investigated across grade-levels, little research exists relating multiplication-fact automaticity (MFA) of developmental mathematics students with course outcomes and/or decisions. The following study explored these topics to reduce the gap in literature, contribute to the body of MFA knowledge, and inform developmental mathematics practices.

Statement of the Problem

Without the necessary skills to pass developmental mathematics courses, fewer students will earn postsecondary degrees (Cafarella, 2014). Bahr (2007; 2013) suggests fewer than one in four students will advance to college-level mathematics due to declining academic performance. Not only does this cost the United States millions of dollars each year through subsidized tuition and government grants, but a failure to remediate many students cripples future generations by becoming a barrier to a more educated and contributing society (Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014; Organisation for Economic Cooperation and Development [OECD], 2012). Multiplication fact automaticity (MFA) may be one of the necessary skills required for developmental mathematics and College Algebra success. This study will explore performance markers of Intermediate Algebra students for differences based on MFA skills.

Purpose of the Study

The purpose of this quantitative, non-experimental study is to understand, through the lens of cognitive load theory (see Kalyuga, Ayres, & Sweller, 2011; Sweller, 1994;

Van Merrienboer & Sweller, 2005), how success in Intermediate Algebra differed based on MFA for students at a small state university in the southwestern United States. Intermediate Algebra is the second course in a two-course developmental mathematics sequence that covers topics included in high school Algebra II. The variable MFA was defined as the score received on a 100-question, timed (i.e., 5 minutes), automated, multiplication-fact assessment for all integers 0-9, which was administered during the first half of the semester of Intermediate Algebra. Comparison variables include the number of attended semesters between enrollment at the research institution and enrollment in a mathematics course; mathematics placement scores; results from each of the first three unit tests; the end-of-course grade; attendance; and course attrition (withdrawal status). Attendance has been found to be one of the best predictors of end-of-course grade (Zientek, Yetkiner-Ozel, Fong, & Griffin, 2013).

Theoretical Framework

Cognitive load theory (CLT) was discovered in the late 1980s by John Sweller while investigating the topic of problem-solving. In the last 25 years, CLT has evolved into a commonly used research tool to inform teaching practices and is well represented in multiple scholarly journals across several disciplines. The premise of the theory is rooted in the limited human working memory (WM) and the necessity to choreograph knowledge between outside stimuli and the unmeasurable capacity of the long-term memory (LTM) to operate in an optimal learning state. Supporters of the theory contend that normal human beings can learn various levels of complex information when the WM is not experiencing a load greater than its capacity (see Kalyuga et al., 2011; Sweller, 1994).

In CLT, two key parts of the human information processing system as well as three types of loads are identified (Kalyuga et al., 2011; Plass, Brunken, & Moreno, 2010; Sweller, 1994). Figure 2.1 contains an illustration of how these attributes are interconnected. According to CLT, the human WM can experience load from three defined categories: extraneous, intrinsic, and germane. An extraneous load is one that is caused by the way material is presented, for example, disorganized presentations and/or literature; rhetoric infused with difficult vocabulary; or oral instruction without visual aids, to name a few. Intrinsic loads derive from the complexity of the materials being acquired. Reducing the information into smaller, more manageable pieces often diminishes the intrinsic load and learning can resume. Finally, a germane load has to do with the learner's investment in acquiring new schema and automation skills, as opposed to using mental faculties on competing stimuli (Plass et al., 2010).

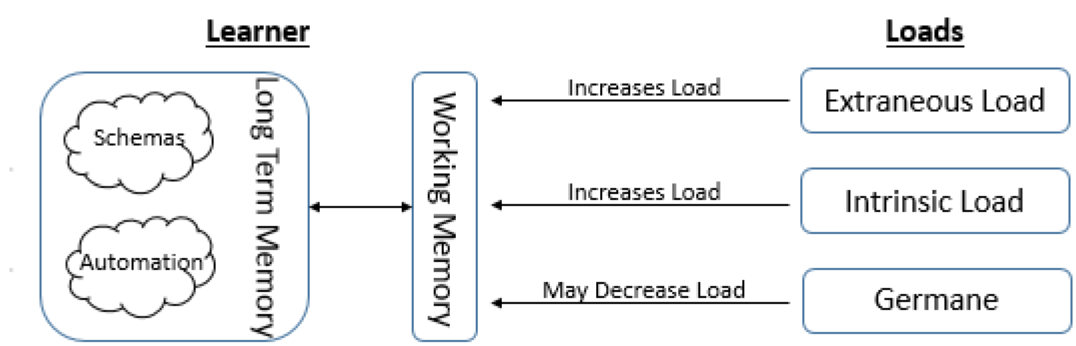

Figure 2.1. A Graphic Representation of Cognitive Load Theory based on Plass et al, (2010).

The tenets of CLT include utilizing long-term memory storage, reducing cognitive load, creating more space in the WM, and facilitating learning ease. When the WM can pass information to the LTM, these tenets are achieved, thus limiting the load. The LTM is able to receive and retain this information if the information can either be attached to an existing schema in the LTM, as described by schema theory (Bartlett, 1932), or learned to a level of automation, akin to driving a car on a familiar route (Kalyuga et al., 2011). Once the information is stored in the LTM it can be retrieved, utilized, and/or built upon without taxing the WM.

Much of Sweller's research in CLT (Kalyuga et al. 2011; Sweller, 1994), has resulted in specific teaching techniques designed to reduce cognitive load and increase learning. Four of the more commonly used methods include, "the goal-free, worked-example, split-attention, and redundancy effects" (Sweller, 1994, p. 308). Each of these methods deliberately inform instructional practices to maximize learning by reducing cognitive load. Additionally, both physiological and self-reported instruments have been developed to measure cognitive load (Plass et al., 2010). While many CLT studies initially focused on teaching mathematics and science, the theory has universal implications and is present in the literature across disciplines with a growing interest in language acquisition (Kalyuga et al., 2011; Plass et al., 2010; Sweller, 1994).

A specific example of cognitive load theory pertaining to MFA is the problem of finding common denominators in fractions. A general rule of thumb about the human WM is that it can handle approximately 7 ± 2 pieces of information depending on capacity, duration, and/or focus (Miller, 1994). Exceeding this parameter results in "cognitive load", a term used to describe a working memory that has been overwhelmed. To solve the fraction problems requires at least five pieces of information including (a) recognizing all denominators; (b) using factor trees or counting-up procedures to identify common multiples; (c) identifying the least of these multiples; (d) using unit fractions to create equivalent fractions; and (e) using the rule of adding fractions with the same denominator (Martin-Gay, 2017). For students who previously have not acquired MFA, the additional steps needed to solve the common denominator problems correctly result in an overload of the WM and interrupt the learning process.

Although the current proposed research does not intend to measure cognitive load, CLT presents an interesting lens through which to view and contemplate the results of the research questions. Certainly, the example of the common denominator problem can be extended to many mathematical topics in Intermediate Algebra, such as factoring trinomials, solving rational equations, and performing operations on radicals, to name a few. Alternatively, CLT may instigate a deeper perspective of measurable, quantitative data from performance assessments, as well as decisions students make toward attendance, attrition, and time-lapses between mathematics courses.

Cognitive Load and Working Memory in Mathematics

The concepts of cognitive load and working memory are influencing research in mathematics instruction (Ayres, 2006; Cooper & Sweller, 1987; Ngu, Chung, & Yeung, 2015; Ngu, Phan, Hong, & Usop, 2016; Pawley, Ayres, Cooper, & Sweller, 2005). Perhaps due to the nationwide high failure and attrition rates of developmental mathematics (Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014), or a personal desire for educators to present material more efficiently and effectively, the interest in improving mathematical skills and programs has been on the rise in recent years (Boylan & Saxon, 2012). Whether an educator chooses to focus on mathematical content, instructional designs, and/or students to address this phenomenon of computational illiteracy, CLT can be applied for greater insight and empirical support.

In CLT, intrinsic load refers to the difficulty that is inherent to the learned concept, which increases with the level of interactivity of elements in the concept (Ayres, 2006; Schmeck, Opfermann, Van Gog, Paas, & Leutner, 2015). Given that working memory has limited capacity, it quickly can be overwhelmed by a complex problem. Ngu et al. (2015) measured cognitive load and instructional efficiency in two test groups of high school mathematics students to compare element interactivity in different methods of solving equations: balance and inverse. For two-step equations, results showed a statistically significant difference between the two methods, favoring the inverse method, which had less element interactivity and cognitive load. Other studies have revealed similar results involving distributive property (Ayres, 2006), pronumeral equation (Cooper & Sweller, 1987), and percent change (Ngu et al., 2016) problems. The research lends itself to the notion that less element interactivity in mathematics reduces cognitive load and increases learning outcomes. Not every mathematical concept can be reduced in element interactivity, but those that can may prove to increase student understanding and performance.

While element interactivity relates to intrinsic load, instructional designs can affect extraneous loads (Kalyuga et al., 2011; Plass et al., 2010; Sweller, 1994). Extraneous loads, which include visual, oral, and multimedia modalities, act as distractors that often are irrelevant to the learning objective and demand attention from the working memory. Pawley et al. (2005) were interested in whether requiring students to check their work during the initial instruction of translating word problems would add to the extraneous load. The work-checking was not an essential component of developing a word-translating schema and, researchers argued that it could be addressed in a follow-on lesson once students were more proficient. Their results showed an overall disadvantage for the work-checking students, with a greater negative impact on higher-level students due to redundancy. In fact, the concept of redundancy increasing extraneous load segues to student affect and its relationship with performance measures. Anxiety, fear, frustration, socio-economic status, and cultural backgrounds potentially could affect extraneous loads (Ashcraft & Kirk, 2001; Kuldas, Hashim, Ismail, & Bakar, 2015), and have been relevant to higher education and specifically to developmental mathematics (Boylan & Saxon, 2012).

Another aspect of CLT involves schemas, automation, and long-term memory. When students previously had developed schemas and automated knowledge of facts in long-term memory, the germane load increases allowing for greater working memory capability. It is not uncommon for higher-level mathematics students to experience a decrease in germane load when the instruction is trivial or, as in the Pawley et al. (2005) study, redundant (Ayres, 2006; Ngu et al. 2015). Abstract concepts inherent to mathematics can be difficult for students without pre-existing schemas, often challenging educators to make lessons relevant, interesting, and valuable. While individualization of mathematics lessons is not always possible, understanding the influence that CLT has on learning and information transfer could revolutionize the way mathematics is taught in the future.

Developmental Mathematics Students' Mathematics Knowledge

Another aspect to understanding the value of specific instructional techniques is to determine the effects of instruction on current K-12 mathematics outcomes. Simply put, what knowledge do developmental mathematics students have from graduating secondary education institutions? Stigler, Givvin, and Thompson (2010) conducted an analysis of 5,830 Mathematics Diagnostic Testing Project (MDTP) results from Santa Barbara City College students during the 2008-2009 school year. The researchers were interested in determining "what students actually understand about the mathematics that underlie the topics they've been taught, including their understanding of the reasons for using known procedures" (Stigler et al., 2010, p. 6). They surveyed 748 developmental mathematics students to search for "evidence that students used reasoning in answering mathematical questions" (Stigler et al., 2010, p. 6). Their findings revealed that students' procedural knowledge was more prevalent than conceptual knowledge, although incorrect timing and application of procedures often resulted in mathematical errors. Second, reasoning skills existed in certain conditions, but were rarely required or developed in prior experiences. Finally, correct answers were consistently given when students could conceptualize the problems (Stigler et al., 2010).

Given the majority of students entering two-year public institutions of higher education enroll in developmental mathematics (Chen & Simone, 2016), Stigler et al. (2010) advocated for new instructional techniques at the post-secondary level to offset the unsuccessful techniques prevalent in the U.S. public school curricula. Currently, developmental mathematics courses tend to over-utilize procedural instruction, consistent with the public-school experience, at the expense of deeper conceptual understanding (Stigler et al., 2010). Through the lens of CLT, an abundance of procedures without schematic connection could result in low completion and retention rates. A basic understanding of developmental mathematics students' prior knowledge and experiences is paramount to informing future redesigns. Procedural and conceptual understanding are not only co-dependent mathematical necessities but require a unique balance that is yet to be defined for classroom practicality and student competency. In a study by Hansen et al. (2015), MFA was found to be a predictor of students' abilities to solve fraction procedure problems. While MFA was not a predictor of students' abilities to understand fraction concepts, the correlation between MFA and fraction concepts was statistically significant and correlated at a noteworthy level (r = .419).

Educational Significance

A high number of students have required remediation before entering college-level mathematics courses (see Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014; Chen & Simone, 2016). The educational significance of this study is to help improve developmental mathematics students' success in mathematics, both at the remedial and college-level, by examining the importance of MFA in their success. If evidence is found that supports differential outcomes in Intermediate Algebra based on MFA skills, educators might want to provide MFA as a prerequisite or formally include it in the curriculum.

Research Questions

1. To what extent do the scores of students with high multiplication-fact automaticity (MFA score ≥.92) differ from those with low automaticity (MFA score < .92) on each of the first three unit tests, end-of-course course grade, attendance, and placement scores?

2. To what extent do the number of semesters that have lapsed between initial enrollment at the institution and enrollment in a mathematics course differ by the two categories of MFA test results?

3. To what extent do the scores of students who withdrew and students who did not withdraw from the Intermediate Algebra course differ in the two categories of MFA test results?

Research Design

This quantitative, non-experimental study was comparative in design (Johnson & Christensen, 2014) because it aimed to identify differences between the two categories of the MFA variable. Additionally, groups were not randomly assigned. Characteristic of a panel study, the same students were assessed for the eight comparison variables. Placement scores and the time lapse variables were retrospective, (i.e., already occurred at the time of the study; Johnson & Christensen, 2014). The results of three separate unit tests, end-of-course grade, attendance, and the decision to withdraw, were longitudinal variables due to the multiple points of collection as time moved forward (see Figure 2.2). Data were collected from each participant during the first six-weeks of the semester, as well as following the last day for complete withdrawal, and at the end of the semester. The researcher conducted quantitative analyses on the data to reveal how the comparison variables differed by MFA test results.

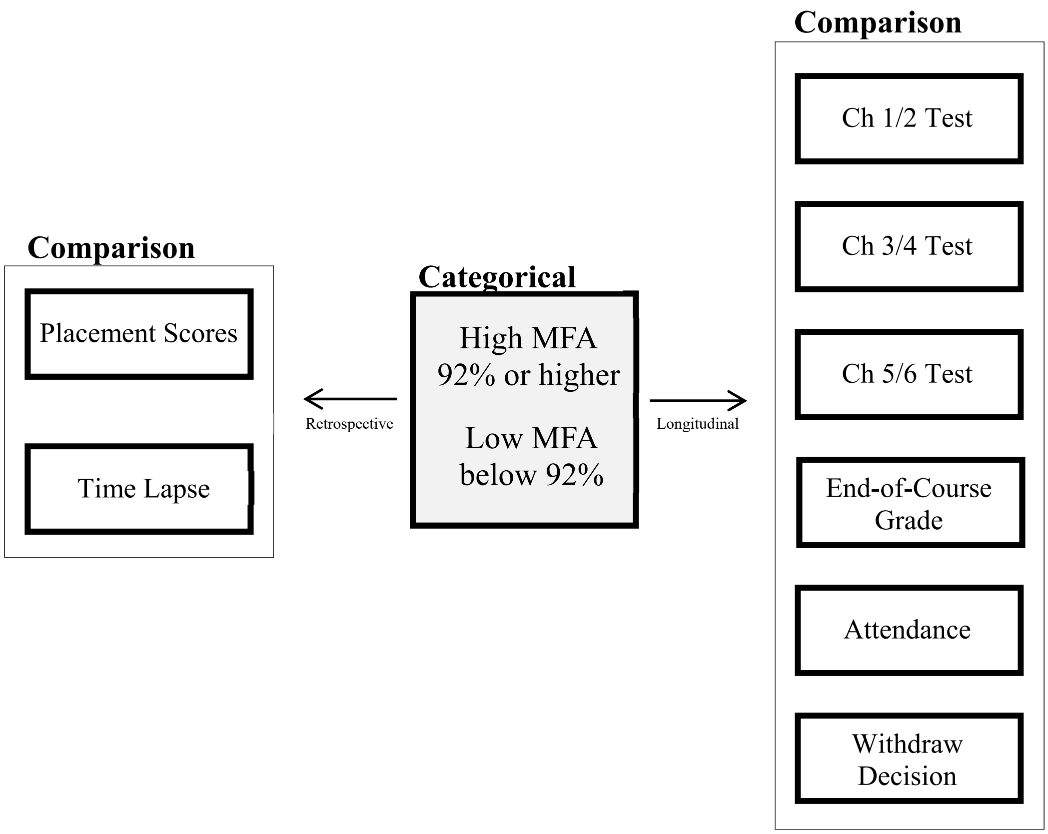

Figure 2.2. Visual Relationship of Variables Examined in the Study.
Note. This model is a comparative design.

Selection of Participants

The participants were chosen using purposive sampling (Johnson & Christensen, 2014) because the specific characteristics of interest to the researcher were students taking Intermediate Algebra at a select institution in the Fall 2017 semester. Intermediate Algebra covers high school Algebra II content and was the highest level of two developmental mathematics courses offered at the participating institution during the time of the study. Placement into Intermediate Algebra requires a C or higher in a prerequisite developmental mathematics course or an appropriate placement test score. Purposive sampling is a non-probabilistic method often chosen for purposes of proximity and availability, as is the case in this research (Johnson & Christensen, 2014). The sampling frame included all students registered for Intermediate Algebra sections at a small, southwestern, public university during the Fall 2017 semester. This open-enrollment institution serves a dual-mission of its original community college charter as well as its recent status change to a 4-year university. The sample was somewhat smaller than the sampling frame due to students choosing not to participate; students who were absent on data collection days; and/or the omission of students registered for Intermediate Algebra sections taught by the researcher.

In the study sample (n = 448) 34.6% were male, 63.8% were female, and 1.6% unknown. According to Wilkinson and the Task Force on Statistical Inference (1999), comparing a convenience sample to its defined population on a large number of variables can strengthen the representativeness of the sample. In this case, the institutional composition of females was slightly lower than the sample at 55.8% and the males slightly higher at 44.2%. The sample consisted of 65.2% White, 17.2% Hispanic, 5.4% African American, 2.2% Native American, and 10.1% other or unknown (institutionally 76.1% White, 11.7% Hispanic, 2.3% African American, 1.1% Native American, and 8.8% other or unknown). Between the ages of 18-24, 88.8% were representative of the sample compared to 62.8% institutionally, and the oldest student in the sample was 66 while the oldest student at the institution was 87. Approximately 83% of the students in the sample reported that Fall 2017 was their first semester at this institution compared to 29.9% of the overall student population. This discrepancy is due to the fact that Intermediate Algebra is an entry-level course and is highly represented by first-year students. The institutional pass rate for this course in recent years has been 52.2% and the attrition rate 22.8%. For the study sample, 60.7% passed and attrition was 12.6%.

Almost 13% of students reported having previously taken Intermediate Algebra at this institution. The most populated field of study was the Health Sciences major at 31.9% followed by 16.3% Humanities; 15.2% Business and Communications; 14.5% General Studies; 10% Science and Technology; 6.0% Education; and 4.5% Arts. The majority of students were single (64.7%), followed by 5.8% married, 2.2% divorced, and 0.7% other or unknown. The majority of students were freshmen (83.3%,), followed by sophomores (9.2%), juniors (4.0%), seniors (2.0%), and unknown (1.6%). All demographic information was obtained from the Institutional Research department and/or from a demographic questionnaire (see Appendix A).

Instrumentation

Students completed two instruments: one to measure MFA and a 1-question background questionnaire. Additional information was gathered from the first three unit tests administered by the course instructors as required in the course curriculum, as well as the results of each student's placement assessment, end-of-course grade, attendance grade, time-lapse, and withdraw status. The study was approved by the Institutional Review Board.

Multiplication fact automaticity (MFA). For the MFA variable, a 100-question, scrambled, electronically delivered, 5-minute timed test of all multiplication facts for integers 0-9 was proctored within the first six weeks of the semester of the Intermediate Algebra courses. The multiplication facts were randomly generated, and the test was administered using MyMathLab, which provided test-modality consistency throughout the course because all Intermediate Algebra lectures, assignments, and tests were delivered through MyMathLab at this institution. MFA tests typically require approximately 1.5 seconds per response (Axtell, McCallum, Bell, & Poncy, 2009), however, this test allows for three seconds per response to accommodate students with disabilities and older students, and because keyboard-entered answers were required.

An interval scale was used to determine the test results as a true zero point cannot measure an absence of MFA (Johnson & Christensen, 2014). The range of scoring was 0-100. The purpose of the measurement was to determine an appropriate distinction between those with high, and low MFA. The median score of 92% was chosen to separate the two categories because median scores separate the bottom half of scores from the top half.

Background questionnaire. A single question was asked on the background questionnaire pertaining to this study: how many times the student reported taking Intermediate Algebra at this institution. The answer categories included one time, two times, three times, or more than three times. This information was used for demographic purposes.

Retrospective variables. The two retrospective comparison variables, placement scores and time lapse, were extracted from student records via the institutional research department. Both variables were intervally scaled. The placement scores used for this study were the highest earned scores on the ACT within the past two years, as those are the scores used for placement purposes. For students who placed into the course using the ACCUPLACER or SAT, equivalent ACT scores were assigned based on the equivalency measures determined by the institution. Furthermore, students who took the SAT prior to March 2016 were assigned a different scale than those after that time period to be consistent with SAT standards and institutional guidelines. Students who placed into Intermediate Algebra by satisfying the prerequisite, Beginning Algebra, as opposed to one of the placement tests were assigned an equivalent ACT score matched with the letter grade earned in the prerequisite course. The researcher created a 10-point scale reflecting the range of ACT scores (13-22) that coincided with placement into Intermediate Algebra. An equivalent 10-point scale was created using letter grade scores 2.0 to 4.0, reflecting admissible grades earned in the previous developmental mathematics course. Time lapse was measured as the number of semesters enrolled at this institution that occurred between initial institutional enrollment and the first enrolled mathematics course. Time lapse data was provided by the institution's research department.

Longitudinal variables. Data from the first three unit tests were collected from MyMathLab by the researcher and the course coordinator. There were two additional unit tests given in this course, but they were not included in this study because they occurred after the final withdrawal date and would have affected the sample size. The first unit test contained 16 free-response questions covering simplifying algebraic expressions; the Addition and Multiplication Properties of Equality; solving linear equations; and problem-solving linear models including translations, percent, area, investment, and mixture problems. The second unit test also had 16 free-response questions. Topics on the test included graphing and finding equations of lines; using the Slope and Point-Slope Formulas; understanding function notation; and solving systems of linear equations using the substitution and addition methods, as well as by graphing. The third unit test was consistent with the first two regarding the number and modality of questions. Rules and definitions of exponents; scientific notation; operations on polynomials; the distributive property of multiplication over addition; factoring binomials and trinomials; and solving quadratic equations including quadratic modeling problems, were among the topics of this test. Every student was given the opportunity to earn two attempts on each unit test by scoring a minimum of 80% on all homework assignments and 100% on notebooks. Scientific calculators were permitted on all assignments and tests. For the purposes of this study, only the highest earned score on each unit test was used, consistent with standards for computing end-of-course grades. These scores were intervally scaled.

Faculty also recorded attendance grades in MyMathLab. Attendance was measured as a percentage of attendance points received out of attendance points possible. The comparison variable of attrition in Intermediate Algebra was measured by using the assigned withdraw (W) grade, as reported by the registrar's office of the institution and/or the instructor, as well as including all students whose last test taken was before the official withdrawal date. A student was considered to have dropped the course in these two instances. Otherwise, the assumption is that a student did not drop the course. This is considered a nominal scale measurement because it is simply a classification of two options (Johnson & Christensen, 2014).

The Intermediate Algebra courses were somewhat homogenous since they were controlled by the mathematics department and were mostly delivered and graded using MyMathLab, a computer-based instructional venue. However, students were required to physically attend instructor-led classes four academic hours per week along with two hours in the math lab. As such, the courses were considered face-to-face. Student end-of-course grades were based on a combined percentage of five weighted categories: attendance (10%), notebooks (5%), homework and test reviews (20%), tests (40%), and the Final Exam (25%). MyMathLab automatically graded everything except the attendance and notebooks, which both had little room for disparities. Attendance and notebook scores were entered into MyMathLab by the instructors and were included in the end-of-course grade computations as per department policy.

Procedures

Prior to conducting this research, full IRB approval was acquired from both the institution housing and the institution overseeing the study; consent forms were collected from the participants (see Appendix B). At the host institution, instructors were given the freedom to conduct skills assessments at the beginning of each semester. The researcher used this opportunity to administer the questionnaire and MFA assessment to all sections of Intermediate Algebra at the main campus. Computer scrambled questions rendered the MFA tests consistent, but unique to deter cheating. Extra credit, equivalent to 0.25% of the overall grade, was offered to students as incentive to take and complete the questionnaire and MFA test.

Participating students were tracked throughout the semester for timely data collection. The researcher was not the instructor for any of the Intermediate Algebra courses participating in the sampling frame. Data were available through the course coordinator, which allowed for ease of accessibility without violating confidentiality. Participants' names were not used or available once the data was extracted.

Data from the MFA test scores were analyzed with the eight comparison variables (placement scores, time lapse, three unit tests, end-of-course grade, attendance, and withdrawal decision) to determine the comparative nature between the two MFA categories. The time needed for this longitudinal study was one semester.

Data Analysis

An independent *t* test was chosen to analyze six of the intervally-scaled comparison variables to see to what extent they differed by MFA test results: scores on the first three unit tests, end-of- course grade, placement score, and attendance. The independent *t* test requires the independent variable to be a grouping variable with which the means of the dependent variable is compared. Several descriptive analyses were conducted on MFA test results, the independent variable, including a grouped and ungrouped frequency distribution displayed both numerically and with a histogram. The researcher chose MFA test results to be the grouping variable and used the median (.92) to separate the variable into two categories, those with high automaticity (MFA score ≥ .92), and those with low (MFA score < .92) because the median is a well-defined separator between the upper and lower halves of a sample.

The six intervally-scaled comparison variables were analyzed for equal variances using Levene's Test for Equality of Variances. In two of the six variables, equal variances could not be assumed: Unit 2 Test and Unit 3 Test. Alternative statistics for not having assumed equal variances were used. The alpha level was set at .05 however, Bonferroni's rule was applicable. Bonferroni discovered that when testing multiple hypotheses, there is an increase in the rarity of the test statistic indicating a rejection of the null. To compensate, Bonferroni's rule reduces the alpha value from the chosen level to the quotient of the alpha and the number of hypotheses being tested, in this case six. Using this correction reduces the likelihood of committing a Type I error of incorrectly rejecting the null (Wilkinson & APA Task Force on Statistical Inference, 1999). Effect sizes and confidence intervals were calculated to address statistical and practical significance.

The chi-squared test was chosen to determine any statistically significant difference between each of the categorical comparison variables, time lapse and withdrawal decision, with the two categories of MFA: high and low. Time lapse was split into four categories: no time lapse between the first semester enrolled at the institution and the first semester enrolled in a mathematics course; one semester lapse; two semesters lapsed; and three or more semesters lapsed. Withdrawal decision was dichotomous: participants did or did not withdraw. In this data set, observed levels were consistent with what we would expect. Most students took their first mathematics course with no time lapse from initial enrollment and subsequent categories decreased in size with the exception of the last category because it bundled three or more semesters of lapse. For the withdrawal variable, 12.5% withdrawal for the sample was lower than the institution's rate of 22.8% but seemed reasonable because the sample excluded students who withdrew during the Fall 2017 semester prior to the data collection period.

Statistical and practical significance. For each test, an alpha level of .05 was chosen a priori, and effect sizes and confidence intervals were considered for both statistical and practical significance (Thompson, 2006; Zientek, Ozel, Ozel, & Allen, 2012; Zientek, Yetkiner, & Thompson, 2010). Effect sizes were calculated on all intervally-scaled comparison variables using Cohen's *d* to assist in interpreting the results without potential confounding caused by sample sizes (Thompson, 2000b), although Cohen (1988) himself put forth his benchmarks "with much diffidence, qualifications, *and invitations not to employ them if possible* [italics added]" (p. 532). Cramer's V was used to calculate the effect sizes for the categorical variables.

Results

Six of the eight comparison variables were analyzed using the independent *t* test. Chi-squared tests were used for the remaining two variables. The sequence of reporting follows this order.

Differences of means. For each of the three unit tests, end-of-course grade, attendance, and placement, the null hypothesis (H_0) stated there were no statistically significant differences between the mean scores of students with high and low multiplication fact automaticity. The alternative hypothesis stated differences between mean scores did exist.

H_0: Mean unit $\text{test}_{\text{high automaticity}}$ – Mean unit $\text{test}_{\text{low automaticity}} = 0$.

H_a: Mean unit $\text{test}_{\text{high automaticity}}$ – Mean unit $\text{test}_{\text{low automaticity}} \neq 0$.

Normality tests, which included quantile-quantile (q-q) plots, were run on the intervally-scaled comparison variables, and normality assumptions for the independent *t* test were satisfied. Table 2.1 contains the *t* test results and descriptive statistics. Levene's test results indicated equal variances could be assumed for all variables except unit tests two and three.

Statistically significant differences existed between students with high and low MFA for each of the three unit tests. In other words, those with high MFA had a statistically significantly higher mean score on each unit test than those with low MFA. Boxplots in Figure 2.3 illustrate the dispersion of data. Boxplots divide data into four equal parts; outliers are identified with the asterisks.

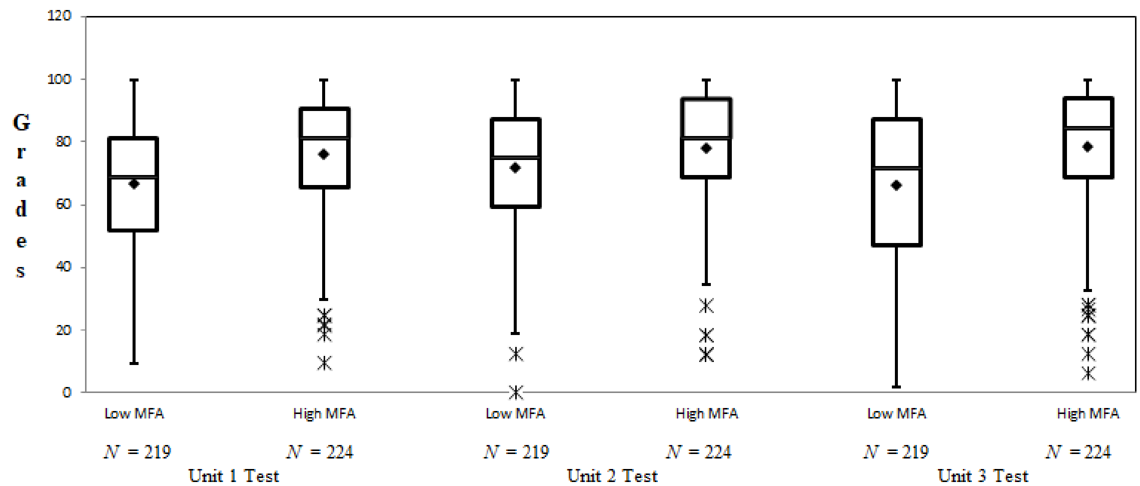

Figure 2.3. Boxplot comparisons for grades disaggregated by MFA category and unit test.
Note. Lines in the middle of the boxes are medians; diamonds = means; asterisks = outliers.

For end-of-course course grade, statistically significant differences also existed between those with high and low MFA. Students with high MFA had a statistically significantly higher mean end-of-course grade than those with low MFA. In contrast, the attendance and placement score variables showed no statistically significant differences between the two levels of MFA. In this case, we failed to reject the null hypotheses that there was no difference between the attendance or placement score means of those with high and low MFA. Recall that students could earn extra attendance points. For attendance, 27 students had over 100% attendance rate; 16 had low MFA and 11 had high MFA. Furthermore, of those 27 students, 17 earned an A; two earned a B; four earned a C; three earned a D; and one earned an F.

Table 2.1

t Test Results for Differences between Students with High and Low Multiplication Fact Automaticity

	t Test Results			Cohen's	Multiplication Fact Automaticity High		Multiplication Fact Automaticity Low		95% CI[a]	
Variable	df	t	*p*	*D*	*M*	*SD*	*M*	*SD*	LB	UB
Unit 1	441	–4.85	<.001	–0.46	76.10	19.43	66.88	20.59	–12.96	–5.49
Unit 2*	430	–3.35	<.001	–0.32	78.27	18.54	71.99	20.64	–9.96	–2.59
Unit 3*	400	–5.43	<.001	–0.53	78.49	20.77	66.06	26.23	–16.93	–7.93
End	442	–3.65	<.001	–0.35	74.04	21.26	66.54	21.99	–11.53	–3.46
Attend	434	–1.12	<.262	–0.11	79.22	21.20	76.87	22.39	–6.45	1.76
Place	386	–1.50	<.134	–0.15	17.19	2.47	16.83	2.32	–0.84	0.11

Note. M = Mean; *SD* = Standard Deviation; CI = Confidence Interval; LB = Lower Bound; UB = Upper Bound. * indicates that equal variances were not assumed.
[a]95% for Mean Differences.

Observed versus hypothesized. The null hypotheses (H_0) was that the proportions of the comparison variables, time lapse and withdrawal decision, did not statistically significantly differ by MFA. The alternative hypotheses were that they did statistically significantly differ by MFA. Time lapse was categorized into no time lapse; one semester time lapse; two semesters time lapse; and three or more semesters time lapse.

Time lapse hypotheses.

H_0: $\text{Proportion}_{\text{high automaticity}} - \text{Proportion}_{\text{low automaticity}} = 0.$

H_a: $\text{Proportion}_{\text{high automaticity}} - \text{Proportion}_{\text{low automaticity}} \neq 0.$

Withdraw hypotheses.

H_0: $\text{Proportion of withdraw}_{\text{high MFA}} - \text{Proportion of withdraw}_{\text{low MFA}} = 0.$

H_a: $\text{Proportion of withdraw}_{\text{high MFA}} - \text{Proportion of withdraw}_{\text{low MFA}} \neq 0.$

Results. There were no statistically significant differences in time lapse composition by MFA, χ^2 (3, N = 438) = 2.00, p =.573, Cramer's V =.068. Similarly, there were no statistically significant differences for withdrawal decision by MFA, χ^2 (1,

$N = 445) = 3.39$, $p = .066$, $\varphi = -0.087$. In both cases, we failed to reject the null hypotheses of no differences. For time lapse, 85.2% of students had no time lapse, 7.3% had a one-semester time lapse, 3.0% had a two-semester time lapse, and 4.6% had three or more semesters time lapse. For withdrawal decision, 12.6% withdrew.

Discussion

Educators across the nation are concerned about the low passing rates in developmental mathematics courses and are seeking ways to increase student success by improving educational elements such as teaching methodologies, learning environments, instructional venues, and noncognitive skills (Bonham & Boylan, 2012; Boylan, 2011; Boylan & Saxon, 2012; Cafarella, 2014). The literature addresses the importance of MFA in multiple settings however, specific studies relating MFA to developmental mathematics students, are not common. The intent of this study was to diminish the gap by exploring whether students with high MFA were more successful in their developmental mathematics courses than those with low MFA.

Even though scientific calculators were permitted, student scores on three unit tests and the end-of-course grade were statistically significantly higher for those with high MFA. The results support existing literature that showed MFA to be a predictor of successful fraction operations (Hansen et al., 2015) and reinforces Wurman and Wilson's claim that "Arithmetic is the foundation. Arithmetic has to be the priority and it has to be done right" (2012, p. 47). These findings suggest that educators can increase success rates in Intermediate Algebra by requiring MFA as a prerequisite. Placement scores; attendance; time-lapse between institutional enrollment and first mathematics course; and withdrawal decisions did not statistically significantly differ by MFA status.

MFA Scores

The sample of developmental mathematics students tended to score well on the MFA test. The median MFA score was .92. Students were categorized as high MFA if their MFA score was at or above the median score and categorized as low MFA if their MFA score was below the median. Statistically significant differences existed between those with high and low MFA for each of the unit tests (see Figure 2.3) and the end-of-grade score (see Figure 2.4). Students with high MFA scored better. The largest differences in means occurred on unit 3 test, over 12 points, while the smallest difference in means, about 6 points, occurred on unit 2 test.

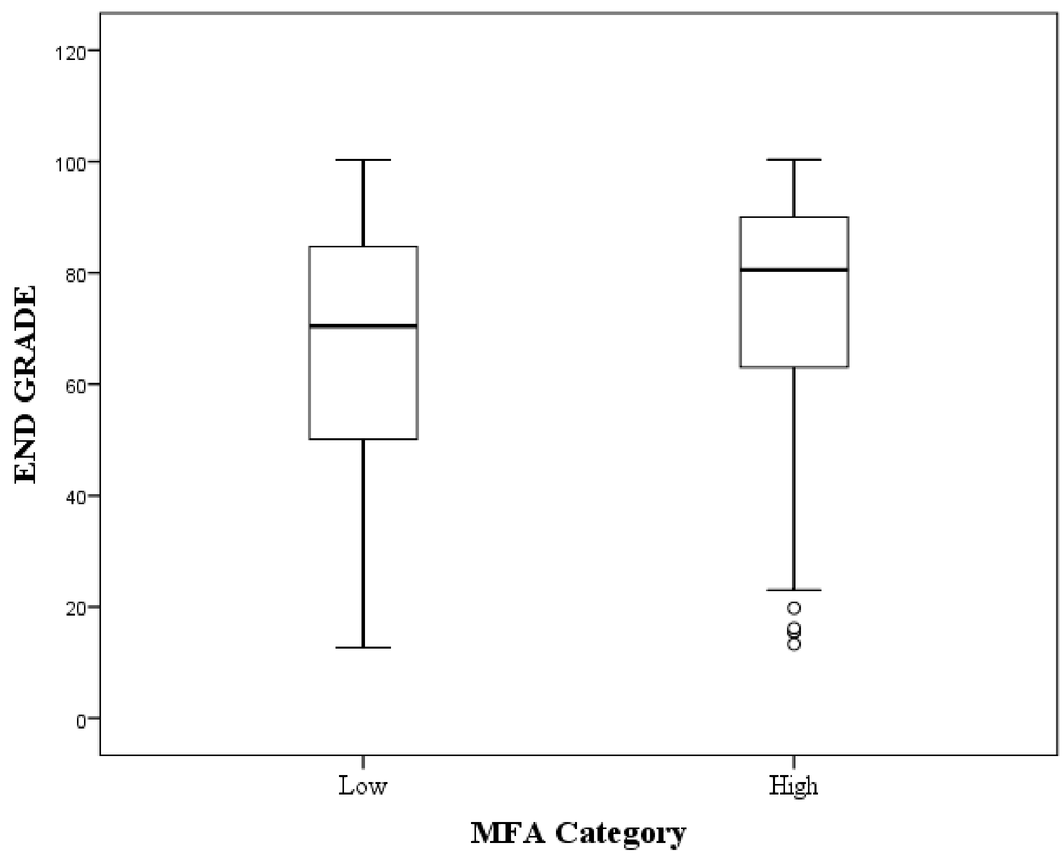

Figure 2.4. Boxplot comparisons for end-of-course grade disaggregated by MFA category. *Note.* Lines in the middle of the boxes are medians; circles = outliers.

Unit 3. Unit 3 test had six questions that required factoring, which is a complex mathematics skill that requires more multiplication-fact dependency. Of the six problems, four were trinomials and two were binomials. The highest score a student could earn without the ability to factor was 62.5%. The mean unit 3 test score for students with low MFA was 66.06% compare to a mean test score of 78.49% for students with high MFA. As seen in Figure 2.3, on the unit 3 test approximately 75% of the students with high MFA scored at or above the median score of students with low MFA. A Cohen's *d* of –.53 means that 70.19% of the high MFA group would be above the unit 3 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 64.61% chance that a person chosen at random from the high MFA group would have a higher unit 3 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012). More outliers were present for grades for students with high MFA compared to students with low MFA, as demonstrated by the asterisks. A qualitative review indicated that six of the eight students who were outliers for the unit 3 test missed 50% or more of the class.

Unit 2. In contrast to unit 3 test, unit 2 test had the least amount of questions requiring factoring; four systems of linear equations questions. If students chose to use the addition method, a greater knowledge of multiplication facts was needed to determine a common multiple; however, students had the flexibility of using the substitution method, which is not multiplication-fact dependent. Regardless, a student could miss all four of those questions and still earn a score of 75%. The mean score on that test for students with low MFA was 71.99%. A Cohen's *d* of –.32 means that 62.55% of the with high MFA group would be above the unit 2 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 58.95% chance that a person chosen at random from the high MFA group would have a higher unit 2 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012).

Unit 1. There was about a 9-point spread between those with high and low MFA on the unit 1 test. Similar to the unit 3 test, as seen in Figure 2.3, on the unit 1 test, approximately 75% of the students with high MFA scored are at or above the median scores of students with low MFA. A Cohen's *d* of –.46 means that 67.72% of the high MFA group would be above the unit 1 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 62.75% chance that a person chosen at random from the high MFA group would have a higher unit 1 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012). Four of the questions involved solving equations with fractions, which required employing multiplication-fact knowledge when finding common denominators. Missing those four questions, however, could still result in a score of 75%. The mean score for students with low MFA was 66.88%, which is a curious discrepancy that would be an interesting continuation of the study. Of the remaining 12 questions, seven were story problems requiring students to translate words into mathematical representations before solving. None of those 12 questions, however, was highly dependent on multiplication-fact automaticity. The mean score for students with low MFA was 66.88%.

End-of-course grade. The cutoff grade for advancing into a college-level course was 70%. For end-of-course grades, there was a statistically significant difference between students with high and low MFA and the difference in mean scores was 7.5 points. More noteworthy, however, was the fact that the mean for those with high MFA (M = 74.04) was considered a passing score for the course whereas a mean for those with low MFA (M = 66.54) was not considered a passing score. Thus, students with high MFA were more likely to advance to a college-level course than students with low MFA. A Cohen's d of –.35, 63 means that 68% of the high MFA group would be above the end-of-course grade mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 59.77% chance that a person chosen at random from the high MFA group would have a higher end-of-grade score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012).

Statistical and practical significance. For each of the unit tests and the end-of-course grades, comparisons of groups with high and low MFA were statistically significant (i.e., $\alpha < .05$) and had practical significance (i.e., effect sizes in medium range). In the case of the end-of-course grade, a 7.5-point difference in means was a sufficient gap to discern whether the students would advance to the college-credit course.

Attendance and placement. No statistically significant differences existed between MFA groups on attendance or placement scores. As noted earlier, both MFA groups utilized the opportunity to earn extra credit in attendance. The difference between the two groups regarding attendance was only 2.35 points. Attendance accounted for 10% of the students' overall grade. It would seem they equally understood the value of attending.

Placement score differences were also negligible at less than one-half of a point difference between the two MFA groups. While there were four placement venues in this sample; ACT, SAT, ACCUPLACER, or successful completion of the prerequisite course, over 97% of the participants took either the ACT or SAT to satisfy their placement requirement. Additionally, the researcher did not do an exhaustive study of the type of questions on the ACT or SAT assessments regarding multiplication-fact knowledge that a student would need to answer correctly to earn a score consistent with the prerequisite requirement for Intermediate Algebra at this institution. Further investigation would be recommended to learn more about any discrepancies between those with high and low MFA.

Attendance and placement score variables had Cohen's *d* effect sizes below 0.2, which generally represents trivial differences not seen by the naked eye. Had they resulted in statistically significant *p* values, the researcher would have considered the practical significance between the two means. In this case, there was no real practical significance and no statistical significance.

Time-lapse and Withdrawal Decision: Observed versus Hypothesized

Time-lapse. The time-lapse variable measured the number of semesters a student waited from initial enrollment at the institution to the first enrollment in a mathematics course. A chi-squared test was conducted to compare the four levels of time lapse: no semesters; one semester; two semesters; or three or more semesters with the two groups of MFA categories. The results showed no statistical significance. Two possibilities could explain this lack of difference.

First, at this institution, freshmen are advised to take mathematics from the onset, to ensure enough time to accommodate possible developmental courses as well as required major-related mathematics courses. For example, an Elementary Education major, if placed into the highest developmental mathematics course upon institutional enrollment, would need four semesters of mathematics before applying to the Education program. A time lapse in mathematics substantially could delay the student's progress toward 4-year degree completion. Based on this advisement practice, many freshmen experience no time lapse, regardless of their MFA classification. For this sample, 85.2% of students had no time lapse. Of those students, 40% did not pass the course and almost half (48.9%) had low MFA. Consequently, if a student's decision to delay taking mathematics was influenced by his or her MFA, it could have been trumped by advisement practices. The second possibility is that students who chose to have a time lapse could have used that time to improve his or her MFA skills. Those who had a time lapse had a 36.3% non-pass rate and 52.3% had low MFA, but we do not know what their MFA rate was upon initial enrollment at the institution. A better test to measure the time lapse variable would have been to collect MFA results upon initial enrollment at the institution, then use those initial scores compared to time lapse.

Withdrawal. Withdrawal decision was also compared to MFA groups and no statistically significant differences existed between the groups. It remains possible that poor MFA skills could contribute to a student's decision to withdraw from Intermediate Algebra; however, several other variables can confound the results and would need to be isolated to get a more definitive picture. In addition, due to personal reasons, students could be influenced by the potential loss of financial aid if dropping a course would reduce their status from full-time to part-time. Other students might choose to persevere in the course to improve their skills for subsequent attempts. Further research is warranted for withdrawal decisions.

Limitations

This study was limited to students attending an Intermediate Algebra course during the Fall 2017 semester at a small, public university in the southwest United States. Participation was voluntary. These results are not generalizable but can contribute to the literature and provide insight for future studies.

Summary

Literature on Cognitive Load Theory (CLT) highlighted the importance of not overloading working memory (WM) while learning various levels of complex information (see Kalyuga et al., 2011; Sweller, 1994). Additionally, intrinsic load, which has to do with the complexity of the learned material, can be reduced with less interactivity of elements, while automation skills reduce the use of mental faculties on competing stimuli (Plass et al., 2010). Cognitive load was not directly tested in this research, but the tenets of its theory support the findings. This study has added to the literature by examining MFA on a sample of developmental mathematics students. The hosting institution will use the results gleaned from this study. A formal MFA pre-requisite will be recommended for Intermediate Algebra students with a minimum score of 92%. This cut-off score is an illuminating outcome of this research as it challenges generalizations about seemingly good scores less than 92%.

Even though calculators were allowed on all assessments, students were more successful in all three unit tests and end-of-course grade having learned multiplication facts to a high-automated level, which likely reduced the load on their working memories and provided an opportunity for complex learning to occur. Specifically, the unit 3 test results had the most profound difference in means between those with high and low MFA and contained factoring trinomial problems, a complex skill requiring multiplication-fact knowledge. Future research is recommended to explore specific relationships between MFA and factoring trinomials.

While MFA will not resolve the developmental mathematics epidemic plaguing our nation, it likely will contribute to student success based on the information learned in this study. Future research is recommended to determine the extent, if any, of predictability MFA has on the comparison variables. Continued research is also recommended to track outcomes of changes made to the Intermediate Algebra course by the hosting institution.

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CHAPTER III

MULTIPLICATION FACTS IN THE CONTINUUM OF SKILLS

This dissertation follows the style and format of *Research in the Schools (RITS)*.

Abstract

This study investigated differences between students' multiplication fact automaticity scores and student competencies on five problems from Intermediate Algebra assessments with a sample of university students. The five types of problems were: (1) linear equations with fractions, (2) system of linear equations, (3) factor by grouping, (4) simplify a rational expression, and (5) simplify a radical expression. Results suggest that prerequisite requirements of multiplication-fact automaticity at or above 94% could increase developmental student success rates in Intermediate Algebra. The findings contribute to the literature on the influence that multiplication fact automaticity can have on student success in developmental mathematics. Educators might consider multiplication fact automaticity in placement, instructional, and curricular practices to better equip developmental mathematics students for success in Intermediate Algebra.

KEY WORDS: Multiplication, Automaticity, Fluency, Multiplication facts.

The debate over requiring multiplication-fact rote memorization has been common at the grade-school level (Carr & Alexeev, 2011; Rouse, 2014). In line with Bloom's Taxonomy, proponents of rote memorization argue the value of basic-skill memorization as a prerequisite to understanding and subsequent mastery of complex problems (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956). However, counting-up techniques, multiplication tables, and calculators provide fodder for the burning question of purpose: why memorize the facts if one can easily compute them?

Developmental mathematics stakeholders find themselves at a similar academic crossroad, but without sound research to weigh-in on the matter. This study was motivated by the gap in research concerning the requirement, or lack thereof, of multiplication-fact automaticity (MFA) for developmental mathematics students. Five problems taken from Intermediate Algebra tests at a small university in the southwest were selected to investigate performance differences in relation to student MFA skills.

Statement of the Problem

Recent studies have identified low passing and retention rates as problematic for developmental mathematics students across the nation (Bahr, 2007; 2013). Developmental mathematics often has been a prerequisite to college-level mathematics courses required for post-secondary degree completion (Bonham & Boylan, 2012; Cafarella, 2014). Subsequently, students lacking basic computational skills are at an increased risk of failing and/or withdrawing from higher education (Bailey, 2009). Nationally, the impact of fewer citizens earning post-secondary degrees can have negative long-term economic, social, and political consequences and can have a negative impact on the U.S. as a member of the global community (Bonham & Boylan, 2012;

Boylan, 2011; Cafarella, 2014; Organisation for Economic Cooperation and Development, 2012). Teaching multiplication skills to a level of automaticity could prove to be a pivotal investment in the developmental mathematics curriculum. The present study examines differences between medians of multiplication-fact automaticity skills and Intermediate Algebra problems to understand the impact multiplication-fact automaticity skills may have on developmental mathematics concepts.

Purpose of the Study

The purpose of this study was to determine whether differences occurred between medians of multiplication fact automaticity (MFA) scores and student competencies on five specific problems from Intermediate Algebra assessments. The sample was obtained at a university in the southwestern United States. This was a non-experimental, quantitative study informed by the theoretical framework of Bloom's taxonomy of cognitive learning dimensions (Bloom et al., 1956; Krathwohl, 2002).

Theoretical Framework

In 1956, Benjamin S. Bloom, along with several measurement specialists in the United States, developed a taxonomy intended to connect student learning outcomes with received instruction (see Krathwohl, 2002). Although the taxonomy underwent a revision in 2001, the focus of the revision primarily was categorical, nominal and dimensional, leaving the principle tenets untouched (Krathwohl, 2002). In both versions, categories of learning are listed based on levels of complexity and concreteness. The most basic and concrete are listed first, graduating to the most complex and abstract. Additionally, each category is considered a prerequisite to achieving mastery of the succeeding category (Krathwohl, 2002).

Bloom's taxonomy, formally referred to as, "The Taxonomy of Educational Objectives" (Bloom et al., 1956), has become a well-known classification tool among educators that has been frequently cited and translated into more than 20 languages. The new version employs a two-dimensional table referred to as the Taxonomy Table and is used for analyzing educational objectives across categories of knowledge complexity and cognitive processes (Krathwohl, 2002). In simpler terms, it is a decision-making tool for the improvement of curricular planning and instructional delivery.

Several concepts in Intermediate Algebra selected for this study relied on the knowledge of multiplication facts as a prerequisite skill. These concepts were solving equations with fractions by clearing fractions; using the addition method for solving systems of linear equations; factoring trinomials; simplifying rational expressions; and simplifying radical expressions. In fact, several intermediate concepts were required as well, which included finding least common multiples, operations with fractions, and factoring integers. Multiplication-fact automaticity skills require the lowest cognitive process of remembering even though educators emphasize multiplication should be taught for both understanding and skills.

As an example, consider factoring trinomials. Within *The Knowledge Dimension*, factoring trinomials would fall under the *Procedural Knowledge* category and correspond to the *Apply* category within *The Cognitive Process Dimension*. Multiplication facts would fall under the *Factual Knowledge* and *Remember* categories, respectively. The skill of remembering multiplication facts, as implicated by the taxonomy, is a subtask inherent to successfully factoring trinomials.

It is important to note that the category, *Remember*, is appropriately titled. It distinguishes itself from other forms of retrieving factual knowledge such as using a chart, using a calculator, or counting-up. Acquiring information from long-term memory allows for detailed focus on complex ideas without laborious and time- consuming distractions (Krathwohl, 2002). Several complex concepts are presented in college mathematics courses which, based on The Taxonomy of Educational Objectives, would require memorizing multiplication facts a prerequisite to achieving mastery. For students who were accurately placed into Intermediate Algebra, a course designed to present and teach objectives that contain multiplication-fact subtasks, it would be a detriment to omit multiplication-fact automaticity from the curriculum or as a prerequisite requirement.

Bloom's Taxonomy and Mathematics

Bloom's taxonomy frequently appears in the literature as a tool to classify mathematics concepts for the purposes of instructional design, learning theory, and assessment (Fan & Bokhove, 2014; Jorgensen, 2010; Lee & Huh, 2014; Roegner, 2013; Woodward, 2004). Critics of Bloom's Taxonomy as a mathematics dimension argue its vague interpretive qualities, which can lead to inconsistent findings and reduced generalizability (Gierl, 1997; Thompson, 2008; 2011). Broadly accepted, however, is the notion that mathematics concepts fall on a continuum of cognitive skills ranging from basic fact knowledge to complex, conceptual, and global competencies, which are more easily quantified when regulated by a well-designed taxonomy.

Thompson (2008; 2011) suggested abandoning Bloom's Taxonomy and replacing Bloom's Taxonomy with mathematics-specific cognitive taxonomies, or standardizing interpretations of dimensions coupled with professional development training in the use of Bloom's Taxonomy for research purposes. Thompson (2011) contributed to these ideas following his research that exposed several inconsistencies in the delineation of lower-order-thinking (LOT) mathematics problems and those requiring higher-order-thinking (HOT) on the North Carolina end-of-course, Algebra I standardized tests. The specific areas of concern included the use of "familiarity" as a point of distinction between HOT and LOT problems; the misinterpretation of real-world problems as being synonymous with HOT; and multiple, broad categories within Bloom's Taxonomy vulnerable to subjective interpretations. Gierl's (1997) work also questioned the efficacy of Bloom's Taxonomy for mathematics research as he discovered that rater predictions and student use of cognitive skills were consistent only approximately 50% of the time; mutual exclusivity was not always guaranteed between domains within the taxonomy; and the predictive accuracy was less precise for lower-level math students. Gierl proposed more detailed mechanisms for psychometric testing of cognitive mathematics skills.

While Bloom's Taxonomy was not designed specifically for the discipline of mathematics, it is still considered an appropriate framework from which more precise dimensions can be extracted to provide cognitive, theoretical, and evaluative instruments for research purposes. Examples in the literature include research on teaching behavior (Lee & Huh, 2014); student performance (Roegner, 2013); content-specific relevance in learning outcomes (Fan & Bokhove, 2014); and historical accounts of mathematics education theories and practices (Jorgensen, 2010; Woodward, 2004). Bloom's taxonomy is globally recognized and frequently cited as a viable tool for cognitive classifications of mathematics concepts. While every researcher must define detailed discriminators to address the intricacies of his or her research questions, Bloom's Taxonomy can provide the underpinnings of the categorization process based on sound theoretical concepts of cognitive learning dimensions.

Multiplication as a Subskill of Factoring

In the taxonomy for this research, multiplication was classified as a subskill of factoring based on common teaching practices that present multiplication instruction prior to factoring instruction (Thornton, 1978). In fact, the definition of a factor is "a number or symbol that divides another number or symbol; when the numbers or symbols are multiplied together, they form a product" (factor, n.d., p.1). In other words, if we know that 3 x 4 = 12, we can then understand that 3 and 4 are both called factors of 12. We can also factor 12 into 3 and 4, or even 6 and 2, or 12 and 1. A logical order exists that defines factoring based on a prior knowledge of multiplication. Further, factoring can be considered similar to dividing, as factor and divisor are synonymous.

The concept of factoring by multiplication suggests that, when factoring, people often rely on memorized multiplication facts (De Brauwer & Fias, 2011; LeFevre & Morris, 1999). Alternative research suggests division (or factoring) can be learned without conceptual knowledge of multiplication skills or facts (Venneri & Semenza, 2011) or that neither influences the other without interference unless they are facilitated as bidirectional operations (Campbell & Robert, 2008).

Educational Significance of the Study

Completion of a college-level mathematics course was a requirement for degree-seeking students at the participating institution as well as many other U.S. colleges. Thus, improving success rates in Intermediate Algebra is related to the achievement of student academic goals. The National Mathematics Advisory Panel (stated “Algebra II correlates significantly with success in college and earnings from employment. In fact, students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation” (U.S. Department of Education, 2008, p. xiii). Hansen et al. (2015) studied various predictors of students’ abilities to conduct fraction concepts and fractions procedures. They found that “researchers and practitioners should explicitly consider students’ attention, working memory, long division, and multiplication-fact fluency skills when developing fraction interventions” (Hansen et al., 2015, p. 47). While multiplication-fact automaticity, which they referred to as multiplication-fact fluency, was a statistically significant predictor of fraction procedures in their regression model, multiplication-fact automaticity did not contribute uniquely in Hansen et al. (2015) model with fraction concepts as the dependent variable. However, Hansen et al. (2015), did report a statistically significant correlation between fraction concepts and MFA ($r = .419$). Thus, educators might consider multiplication-fact automaticity in placement, instructional, and curricular practices to better equip developmental mathematics students for success in Intermediate Algebra.

Currently, the institution in this study does not specify MFA skills as a placement qualifier; nor are these skills included in the curriculum of the course. Time constraints, availability of calculators, and underlying assumptions of effective student study practices are typical justifications for maintaining existing practices, as indicated by the former department chair and Vice-President of Academics (D. Dillingham-Evans, personal communication, July 14, 2014). As the developmental mathematics failure and attrition crisis is not unique to this institution (Bahr, 2013; Bonham & Boylan, 2012;

Cafarella, 2014), findings from this study could contribute to future mathematics educational practices nationwide.

Research Question

To what extent did students who correctly answered each of five problems from Intermediate Algebra tests differ on their median multiplication-fact automaticity scores from those who did not answer correctly? For statistically significant differences, effect sizes were provided to understand the magnitude of the differences.

Research Design

This quantitative study investigated differences in multiplication-fact automaticity scores between two groups of students: those who answered the problems correctly and those who answered incorrectly (see Figure 3.1). The independent variables were not manipulated, and the sample was not random, which rendered this a non-experimental study (Johnson & Christensen, 2014). Data collection occurred on six separate occasions during the Fall 2017 semester, consistent with a longitudinal design.

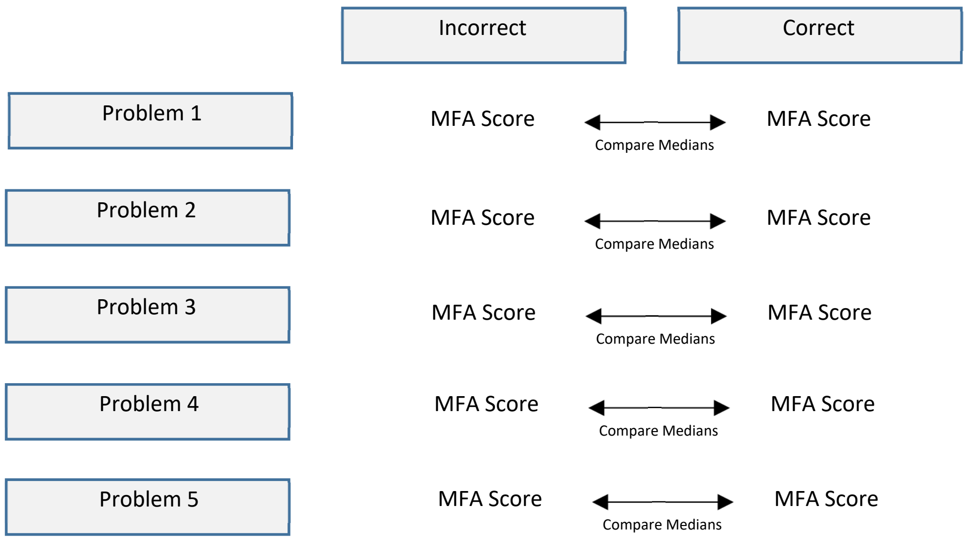


Figure 3.1. Illustration of Models examined in Research Question.

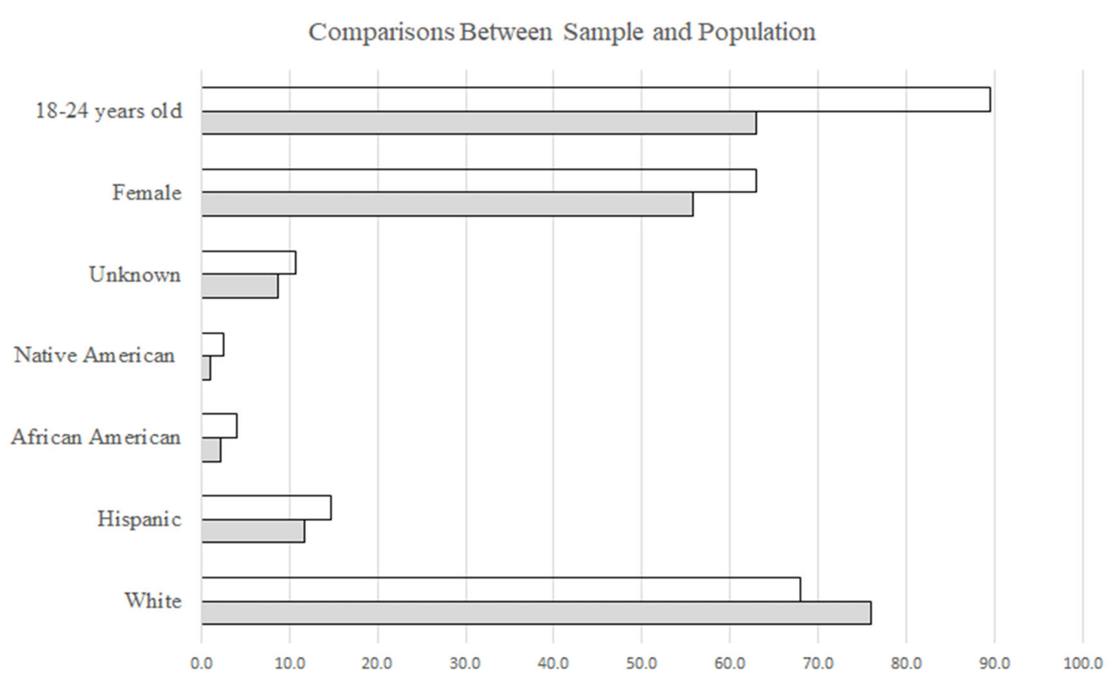
Note. These examine the extent to which the medians of each category of the independent variables differ.

Selection of Participants

The sampling frame for this research consisted of all adult students registered for Intermediate Algebra at the main campus of a small, public university in the southwestern U.S. during the Fall 2017 semester. Intermediate Algebra was the highest level of two developmental mathematics courses offered at the participating open-enrollment institution that served both a community college, as well as a 4-year university mission. Purposive sampling was elected due to high failure (47.8%) and withdrawal rates (22.8%) of Intermediate Algebra students at this institution, and for efficiency of resources (Johnson & Christensen, 2014). Scores from 83 students who did not answer all five test problems under investigation were removed from the data, as well as scores for six students enrolled in sections taught by the researcher.

Participant characteristics were obtained through the institutional database to develop the sample's demographic blueprint. According to Wilkinson and the APA Task

Force on Statistical Inference (1999), comparing several variables of the population to the sample provides evidence of the sample’s representativeness of the population. Figure 3.2 contains the demographics of the university enrollment data and the sample (n = 365). The sample and population were comparable on all demographics except age. The majority of students were White, followed by Hispanic, Unknown, African American and Native American. The sample had a higher percentage of 18-24 year-old students than the population. The oldest student in the sample was 66 years; compared to 87 at the institution. Most students in the sample were freshmen (81.9%), although all classes were represented (2.5% seniors, 4.1% juniors, 10.1% sophomores, and 1.4% unknown).



	White	Hispanic	African American	Native American	Unknown	Female	18-24 years old
Sample	67.9	14.8	4.1	2.5	10.7	63.0	89.4
University Population	76.0	11.7	2.3	1.1	8.8	55.8	63.0

Figure 3.2. Demographic Comparisons. *Note.* n = 365; *Numbers represent percentages.*

Instrumentation

Six instruments were used for this study: a multiplication-fact automaticity test and five tests that were part of the course curriculum. All tests were administered through MyMathLab, which was the instructional delivery system already in use for the Intermediate Algebra courses. These courses closely resembled a flipped instructional model as video lectures were to be watched outside of class via MyMathLab and in-class activities included traditionally assigned homework problems that were also delivered through MyMathLab.

Multiplication-fact automaticity. The MFA test was a 100-question, free-response assessment administered during the first five weeks of the Fall 2017 semester and consisted of all multiplication facts for integers 0-9. Participants were given 5 minutes to complete the test, and MyMathLab provided instant results. The problems were presented in random order as to prevent cheating and persistent use of the counting-up technique. Calculators were not permitted.

Tests. Five tests were administered during the Intermediate Algebra course, as per the curriculum, and covered content from the first ten chapters. To measure skills pertaining to multiplication-fact knowledge, a single problem from each test was selected as the independent, dichotomous variable for each analysis (see Table 3.1). These problems were not revealed to the participants, instructors, or proctors. All five problems were characterized by potentially requiring the subskill of multiplication-fact knowledge to correctly solve. MyMathLab generated random integers within each problem to render them unique for each student while maintaining the same instructions and problem-type. Thus, students did not answer the exact problems illustrated in Table 3.1, rather similar versions with modified integers. Data were made available by the department analyst and compiled by the researcher for this study. For each test, participants were permitted non-graphing calculators, were given 50-minutes for completion, and could change previously answered problems during the 50-minute test session. Additionally, participants could earn a second attempt for each test based on satisfying specific prerequisite requirements. Consistent with grading practices at the institution, only the test with the highest score for each unit was considered.

Table 3.1

Overview of Independent Variables (Problems)

Problem	Problem Type	Example	Multiplication Fact Sub-skill
1*	Linear equation with fractions	$\frac{7}{9}x - \frac{1}{3} = 2$	Find the LCD of all terms
2*	System of linear equations	$\begin{cases} 2x - 5y = 2 \\ 8x - 20y = 4 \end{cases}$	Find the LCM of the coefficients of one variable
3	Factor a trinomial	$4x^2 - 11x + 7$	Find factors of the products of the first and last coefficients whose sum is the middle coefficient.
4	Simplify a rational expression	$\frac{4x - 32}{x^2 - 8x}$	Find the greatest common factor of the numerator and denominator and simplify.
5	Simplify a radical expression	$5\sqrt{125} - 2\sqrt{28} - 7\sqrt{45}$	Factor each radicand to identify perfect squares and reduce.

Note. Students were not penalized for using alternative methods.
* indicates an alternative method had also been taught in the course which did not require a multiplication fact sub-skill.

Procedures

Upon IRB approval, participants read and signed a consent form outlining the details and purpose of the study (see Appendix B). The consent form was available on MyMathLab and could be completed electronically during the first five weeks of the Fall 2017 semester. Following consent, participants could earn 0.25% extra credit by completing the multiplication-fact automaticity skills assessment, administered in the mathematics proctor center. Each participant had a single opportunity to take the multiplication-fact automaticity test without the use of calculators, learning aids, or help from peers. The multiplication-fact automaticity skills assessment did not count toward students' course grades.

The remaining five-data points were collected from unit assessments that were scheduled approximately every two weeks throughout the semester. The last test for data collection was administered in the 13th week of the semester. The researcher was given access to all six assessments and categorized the data by participant and variable. Encryption procedures were used to protect individuals' privacy and to reduce potential bias. The data collection process was completed in one academic semester.

Data Analysis

Parametric assumptions were tested (see Results section) and both statistical and practical significance were addressed. For each analysis, an alpha level of .05 was chosen a priori. Eta-squared (η^2) effect sizes and confidence intervals were also considered (Thompson, 2006; Zientek, Ozel, Ozel, & Allen, 2012; Zientek, Yetkiner, & Thompson, 2010). Effect sizes assist in interpreting results without potential confounding effects caused by the sample sizes that can occur with *p* value calculations, as well as provide the magnitude of the effect (Thompson, 2000b). Cohen (1988) invited authors not to interpret effect sizes with the rigidity of *p* values.

Results

Preliminary Analysis

For the independent *t* test, the dependent variables should be continuous, and the independent variables should be categorical (Cohen, 1988). In each case, the continuously scaled MFA was chosen as the dependent variable with scores ranging from 0-100. Test problem answers were the categorical independent variables that were split into two groups: correct or incorrect. The independence of observations assumption was satisfied in each case because a single sample could not exist in both categories; either the answer was right, or it was wrong, but not both.

Because the *t* test is a means test, cases that deviate markedly from the rest (extreme outliers) unduly skew the mean in that direction therefore, it is important in the data cleaning stage to identify and deal appropriately with those cases. Boxplots were created to reveal extreme outliers, defined as being more than one and a half box-lengths above or below the edge of the box (Sullivan, & Verhoosel, 2013). Problems 2, 3, and 4 each contained a single, extreme outlier. The culprit was a student who scored incredibly low on the MFA (22%) but correctly answered problems 2, 3, and 4. Overall results were not altered by removal of this outlier; the outlier was left in the data before testing additional assumptions.

Normality and homogeneity of variances. The independent *t* test has two additional assumptions for the independent variable: normal distributions and equal variances. An account of testing those assumptions for MFA scores follows below and is disaggregated by test problem.

Problems 1, 2, 3, 4 and 5. On problem 1 (linear equation with fractions) for each group, the dependent variable MFA score satisfied homogeneity of variances based on Levene's test but failed the normality assumption based on Shapiro-Wilk's test. Because equal variance assumptions were met but normality assumptions failed, the Mann-Whitney U test was conducted to determine statistically significant differences between MFA median scores for question 1 by group. The Mann-Whitney U test is the non-parametric version of the independent *t* test, Furthermore, comparison of median MFA scores was more appropriate than means because a visual inspection of the frequency histograms revealed distributions of MFA scores were skewed but similar for each group. According to Sheskin & Sheskin (2004), it is reasonable to look at the medians when the distributions are similar (including similarly skewed). Tests on assumptions had equivalent results for problems 2 (system of equations), 4 (simplify a rational expression), and 5 (simplify a radical expression).

Problem 3. On problem 3 (factor trinomial), initially both groups of MFA scores failed Shapiro-Wilk's normality test and Levene's test of equal variances. The data were trimmed by removing 12 participants (approximately 3%) from the sample (i.e., six from the top and six from the bottom). This trimming technique allowed the statistic to be more robust by reducing influences from outlier scores (Thompson, 2006) while leaving the median unaffected. The new sample (n = 353) still failed the normality assumption but satisfied equal variances. With the trimmed data, the assumption tests and comparisons of distributions were similar to the other test problems; therefore, a Mann-Whitney U test was conducted.

Differences in Multiplication Fact Automaticity Scores

Mann-Whitney U tests were conducted for all five problems to determine if there were differences in multiplication-fact automaticity (MFA) median scores between those who answered correctly, and incorrectly. Visual inspection of histograms indicated that the distributions of the MFA scores for the two categories were similar in each analysis. The null hypothesis (H_0) stated there were no statistically significant differences between the median MFA scores of students who answered the problems correctly versus incorrectly. The alternative hypothesis stated differences (H_a) between median scores did exist. Tables 3.2 and 3.3 summarize the results.

H_0: Median MFA $\text{score}_{\text{correct}}$ – Median MFA $\text{score}_{\text{incorrect}} = 0$.

H_a: Median MFA $\text{score}_{\text{correct}}$ – Median MFA $\text{score}_{\text{incorrect}} \neq 0$.

Problem 1 (Linear equation with fractions). No statistically significant differences existed on median multiplication-fact automaticity scores between the groups that incorrectly (*Mdn* =.91) and correctly (*Mdn* =.94) solved a linear equation with a fraction, $U = 12329$, $z = –1.533$, $p = .125$. The decision was to fail to reject the null hypothesis of no differences in median MFA scores.

Problem 3 (Factor trinomial). Statistically significant differences existed on median multiplication-fact automaticity scores between the groups that incorrectly (*Mdn* =.89) and correctly (*Mdn* =.94) factored a trinomial, $U = 10169$, $z = –2.189$, $p =$

.029, η^2 = .014. The decision was to reject the null hypothesis of no differences in median MFA scores.

Question 5 (Simplify rational expression). Statistically significant differences existed on median multiplication-fact automaticity scores between the groups that incorrectly (*Mdn* =.85) and correctly (*Mdn* =.95) simplified a rational expression, U = 6404, z = –3.104, p = .002, η^2 = .026. The decision was to reject the null hypothesis of no differences in median MFA scores.

Table 3.2

Mann-Whitney U Median Test Results for MFA Differences between Students Who Answered Incorrectly and Correctly for Each Problem

	Mann-Whitney U Results			Effect	MFA Score for those answered Incorrectly	MFA Score for those answered Correctly
Problem	U	Z	*P*	η^2	*Median, 95% CI*	*Median, 95% CI*
Linear equation w/ fractions	12329	–1.533	<.125	n/a	.91 [.86, .95]	.94 [.90, .96]
System of linear equations	9234	–1.105	<.269	n/a	.92 [.85, .96]	.93 [.90, .96]
Factor trinomial*	10169	–2.189	<.029	0.014	.89 [.83, .95]	.94 [.91, .96]
Simplify a rational expression	6404	–3.104	<.002	0.026	.845 [.78, .92]	.95 [.91, .96]
Simplify a radical expression	11562	–3.324	<.001	0.030	.885 [.86, .92]	.95 [.92, .97]

Note. CI = Confidence Interval. *indicates approximately 3% trimmed data.

Table 3.3

Mean Test Results for MFA Differences between Students Who Answered Incorrectly and Correctly for Each Problem

	MFA Score for those answered			
	Incorrectly		Correctly	
Problem	*M, 95% CI*	*SD*	*M, 95% CI*	*SD*
Linear equation w/ fractions	85.33 [82.3, 88.3]	15.63	87.98 [86.3, 89.7]	13.76
System of linear equations	84.76 [80.7, 88.8]	16.70	87.77 [86.2, 89.3]	13.74
Factor trinomial*	85.20 [82.3, 88.1]	14.15	88.78 [87.3, 90.3]	12.30
Simplify a rational expression	81.86 [77.6, 86.2]	16.04	88.18 [86.6, 89.7]	13.84
Simplify a radical expression	83.90 [81.0, 86.8]	16.08	88.84 [87.2, 90.5]	13.17

Note. M = Mean; *SD* = Standard Deviation; CI = Confidence Interval. * indicates approximately 3% trimmed data.

Discussion

In recent years, developmental mathematics has become a focal point among students, educators, administrators, and policy-makers due to the high rates of attrition and failure that has resulted in it becoming a barrier toward degree completion (Bahr, 2013; Bonham & Boylan, 2012). If rigor is to be maintained and mathematics literacy is to remain a valuable component of higher education, the identification of flaws in curricular expectations, student preparedness, and/or instruction, must be a priority. Students need to be equipped with the means to both engage and succeed in developmental mathematics. The literature contains several topics that contribute to understanding low success in developmental mathematics including, but not limited to, student affect, instructional designs, study skills, and placement procedures (Bonham & Boylan, 2012; Cafarella, 2014). However, very little exists concerning the impact multiplication-fact automaticity, or the lack thereof, may have on student performance. The purpose of this study was to investigate differences of multiplication-fact automaticity between groups of students who either could or could not solve specific problems selected from tests administered in an Intermediate Algebra course.

Multiplication Fact Automaticity by Group and Problem Type

Five categories of problems were investigated in this study: (1) linear equation with fractions, (2) system of linear equations, (3) factor by grouping, (4) simplify a rational expression, (5) simplify a radical expression. Statistically significant differences existed on multiplication-fact automaticity median scores between groups for questions 3, 4, and 5, and students who correctly answered those questions had median scores at or above 94%. These findings support existing literature linking multiplication-fact retrieval to factoring (De Brauwer & Fias, 2011; LeFevre & Morris, 1999) and suggest the possibility that prerequisite requirements of multiplication-fact automaticity at or above 94% could increase student success rates in Intermediate Algebra. In contrast, no statistically significant differences existed on multiplication-fact automaticity scores for problems 1 and 2. For solving those two problems, students could have used alternative techniques that would not have benefitted from previously having achieved multiplication-fact automaticity. The next sections further explain results by problem-type.

Problem 1: Linear equation with fractions. This problem was designed to evaluate students' abilities to solve arithmetical equations of the form $ax \pm b = c$ that contained fractions (see Table 3.1). The reason this problem type is called an arithmetical equation is because finding the solution requires "only arithmetical operativity with numbers" (Filloy & Rojano, 1984, p. 53; as cited in Kieran, 1992). No statistically significant differences existed in median multiplication test scores among the two groups of students (p = .125), and 71% of students answered this problem correctly. The median score for those who answered incorrectly was .91, versus .94 for those who answered correctly.

Students were taught two methods for solving this type of arithmetical equation; one method did not require multiplication skills and the second method did. The first method did not require multiplication skills as both inverse operations could be computed on a scientific calculator with fraction functions. This type of calculator was permitted and encouraged in the classroom, as well as on tests. To solve with method one, inverse operations were used to isolate the variable. For the example in

Table 3.1, the student would use the calculator first to add 2 + 1/3 to get 7/3, then to divide that answer by 7/9 to get 3.

The second method taught for solving the equation was clearing fractions. The importance of teaching this method was to encourage students to practice the technique in anticipation of more difficult fractional equation problems in future chapters. The identification of problems with arithmetic and clearing fractions dates back to an article published in 1930 that identified those errors as common in ninth grade algebra (Whitcraft, 1930). However, Jones, Zientek, Sharon, and Swarthout (under review) found that few prospective elementary teachers attempted to solve arithmetical equations by clearing fractions. To employ the clearing fraction method, students first need to determine the LCD of all terms in the equation (in this case, the LCD was 9), then multiply all terms by 9 to clear the fractions. The resulting equation would be $7x - 3 = 18$. This is an arithmetical equation that is much simpler and can be solved by adding 3, then dividing that answer by 7. Finding the LCD is a process that requires knowledge of multiplication, division, and factoring. Students entered their responses on MyMathLab, a computer learning site, without indication as to which method was chosen. Future research would benefit from enforcing the second method to determine any connection between multiplication-fact automaticity and solving equations with fractions by clearing fractions.

Problem 2: System of linear equations. The second problem asked students to solve a system of linear equations (see Table 3.1). Again, no statistically significant difference existed ($p = .269$) between the median multiplication-fact automaticity scores of those who answered incorrectly (.92) compared to those who answered correctly (.93), and 81% answered the problem correctly.

Similar to problem 1, students were taught two solving methods: one method did not require multiplication skills and the second method did require multiplication skills. The first method did not require multiplication skills because inverse operations could be calculated with scientific calculators. This method was the substitution method, which employed inverse operations on one equation to write one variable in terms of the other and then substitute that answer into the second equation to solve for a single variable. Finally, that answer could be back-substituted to find the other variable. As in the first method of problem 1, this technique could be managed with calculations performed on a scientific calculator; thus, the use of multiplication-fact automaticity could be avoided.

The second method was the addition method, which was consistent with the problem's instructions. In this case, coefficients of one of the variables were to be manipulated through a multiplicative process, similar to finding common denominators (Carnine, Jitendra, & Silbert, 1997; Feldman, 2014; Harel & Confrey, 1994; Zazkis & Campbell, 1996). The resulting products would be least common multiples (LCMs) of each other, having opposite signs. The equations could then be added, and the manipulated terms essentially would cancel out, leaving a single variable and a single equation to solve. This answer would then be back-substituted to find the other variable. The process of finding an LCM is equivalent to finding an LCD (see problem 1) and is heavily dependent on multiplication-fact skills.

Problem 3: Factor by grouping. As seen in Table 3.1, the instructions for this problem were to factor the polynomial by grouping. Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups (p = .029). The median score for the group that incorrectly factored by grouping was .89 compared to a median score of .94 for the group that correctly answered the problem; 74% of students correctly answered this problem. The effect size implies 1.4% of the variability in ranks was accounted for by the independent variable (i.e., groups incorrect or correct).

Only one technique was taught to solve this type of problem, and it required multiplication-fact skills to solve. To solve, students had to multiply the leading coefficient (in this case, 4) by the constant coefficient (7) to get a product of 28. Then students had to find factors of this product whose sum was – 11 (the coefficient of x). In this case, the factors were – 4 and – 7. Their product was 28 and their sum was – 11. Students would use the factors in the factor-by-grouping algorithm to re-write the initial problem as 4-terms instead of three. At that point, the algorithm required factoring out the greatest common multiple (GCM) from the first two terms, then again from the second two terms, and finishing the algorithmic steps. Finding factors, as well as factoring out the GCMs, requires multiplication-fact skills not easily by-passed with scientific calculators.

Problem 4: Simplify rational expression. Problem 4 contained a rational expression that needed to be simplified (see Table 3.1). Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups (p = .026). The median score for the group that incorrectly factored by grouping was .85 compared to a median score of .95 for the group that correctly answered the problem; 85% of students correctly answered this problem. The effect size implies 2.6% of the variability in ranks was accounted for by the independent variable (i.e., groups incorrect or correct).

Only one technique was taught to simplify rational expressions. Multiplication-fact skills were required to solve. The process required factoring out the GCM from both the numerator and denominator, then reducing any common factors to achieve a simplified result. In the example given, the GCM of the numerator was 4 because it was a factor of $4x$ and –32. Once 4 was factored out, the remaining binomial was $x - 8$. In the denominator, x was the GCM, leaving $x - 8$ as the remaining binomial. Because the numerator and denominator contained the same factor ($x - 8$), it could be cancelled because any factor divided by itself equals 1. The simplified solution to this problem was the fraction $4/x$. As explained in problem 3, finding a GCM requires multiplication-fact skills not easily circumvented with scientific calculators.

Problem 5: Simplify radical expression. The instructions for problem 5 were to add or subtract radical expressions. Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups ($p = .001$). The median score for the group that incorrectly factored by grouping was .89 compared to a median score of .95 for the group that correctly answered the problem; only 67% of students correctly answered this problem. The effect size implies 3% of the variability in ranks was accounted for by the independent variable (i.e., group incorrect or correct). No other technique was taught for this problem.

Students were taught one method and calculators capable of simplifying radicals were not permitted. Thus, multiplication-fact skills were necessary. Students were taught to only add or subtract like terms. However, to discern whether like terms existed in this problem students had to first simplify each radical. For the first term, 125 was the radicand and could be factored into 25 and 5. These two factors were chosen because 25 is a perfect square and could be removed from the radical. The square root (sqrt) of 25, which is 5, could be multiplied together with the 5 that was in front of the radical sign. The product became 25, on the outside, and the simplified term was 25 sqrt 5. Similarly, the radicand of the second term, 28, could be factored into 4 and 7. These factors were chosen because 4 is a perfect square whose square root is 2, leaving only 7 inside the radical. The 2 from the radical could be multiplied with the 2 outside of the radical to result in 4 sqrt 7. The third term could be simplified to 21 sqrt 5. The first and last terms both resulted in having 5 as the radicands, meaning they were like terms and could be combined. The final answer was 4 sqrt 5 – 4 sqrt 7.

Overview of statistical and practical significance. For the three problems that required multiplication-fact skills to solve, statistically significant differences existed between median multiplication-fact automaticity scores (i.e., $\alpha < .05$). These were problems 3, 4, and 5, which respectively were factor by grouping, simplify a rational expression, and simplify a radical expression. It is meaningful to consider the practical significance of the ranges between incorrect and correct categories for each of problems 3 (.89 versus .94), 4 (.85 versus .95), and 5 (.89 versus .95) that supports the notion of targeting a multiplication-test score in the low .90s as a prerequisite course requirement.

Limitations

This study was limited to a volunteer group (*n* = 365) of Intermediate Algebra students 18 years of age and older from a small university in the southwest United States during the Fall 2017 semester who were not enrolled in sections taught by the researcher. The results are not generalizable. The sample only included students who completed all five tests, consequently eliminating many of the academically weaker students who stopped attending the course mid-term. Additional limitations included the researcher's inability to discern which technique students used to answer the first two test problems, each of which formally included two instructional means for solving.

Summary

Bloom's taxonomy is widely accepted as a tool to regulate levels of complexity and concreteness (Krathwohl, 2002). Educators frequently use it as a decision-making tool to improve curricular planning and instructional delivery (Fan & Bokhove, 2014; Jorgensen, 2010; Lee & Huh, 2014; Roegner, 2013; Woodward, 2004). Mathematical concepts range in complexity from basic fact knowledge to multi-layered, abstract problems that challenge the most gifted of mathematicians. Classifying the mathematical continuum of facts, algorithms, and problem-solving within a specific learning objective can reduce problematic concepts by illuminating prerequisite categories which may have been overlooked, or underemphasized (Fan & Bokhove, 2014; Krathwohl, 2002; Roegner, 2013).

This research contributes to the literature in two ways: its consistency with the theoretical framework of Bloom's taxonomy, which suggests mastery at each category is dependent upon mastery in the preceding category (Krathwohl, 2002), and by reducing the gap addressing multiplication-fact automaticity and its influence on student success in developmental mathematics. Problems 3 (factor by grouping), 4 (simplify a rational expression), and 5 (simplify a radical expression) resulted in statistically significant differences between median multiplication-fact automaticity test scores for those who answered incorrectly versus correctly and multiplication-fact skills were required for solving, even though calculators could be used. However, for problems that students could choose a method that did not require multiplication-fact skills, no statistically significant differences existed in median multiplication-fact automaticity test scores. Problem 4, a rational expression problem, had the biggest disparity in multiplication-fact automaticity test scores: a median of .85 for those who answered incorrectly versus .95 for correctly. The problem exclusively required factoring and division, both successor categories of multiplication.

These findings imply a suggested multiplication-fact automaticity mastery threshold greater than 90% and a curricular shift to either include automaticity-level multiplication-fact skill development within the Intermediate Algebra class or as a prerequisite requirement. Future research is recommended to explore the extent to which developmental mathematics courses employ factoring concepts and skills, as well as the dependency factoring has on multiplication-fact automaticity. A meta-analysis of multiplication-fact requirements for developmental mathematics courses across the nation could initiate a conversation of generalizability.

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CHAPTER IV

THE LIVED EXPERIENCES OF INTERMEDIATE ALGEBRA WITHDRAWAL STUDENTS

This dissertation follows the style and format of *Research in the Schools (RITS)*.

Abstract

Symbolic interactionism has been used to gain understanding of human perceptions in the classroom. Students' perceptions that influence their choice to withdraw from a developmental mathematics course could inform instructional and administrative practices. This study explored, through eight personal interviews, the lived experiences of students who withdrew from a technology-based, developmental mathematics course at a small university. Interviews were transcribed verbatim resulting in 123 significant statements. Six themes emerged: student goals, false course-expectations, the decision to withdraw, mathematics experiences, strategies for success, and mathematics self-efficacy. Understanding participants' experiences may prompt change in student-teacher engagement, marketing of resources, understanding the implications of student self-efficacy, personalizing basic-skill improvement strategies, and fostering an appreciation for overwhelming demands inherent of some developmental mathematics courses.

KEY WORDS: Developmental math, Attrition, Withdraw, Self-efficacy.

A student in her 30s with a family and a job who withdrew from an Intermediate Algebra course wrote about her experience:

> I was so far behind with trying to get through all the assignments. I had five other classes besides this one, so I was trying to do good in all of those other classes, and I would spend hours, and hours, and hours at night trying to do homework for this one. And so, when I got behind on one assignment, it kept piling up because then I would try to finish to get the 80%. By the time I got that done, the next one was late. And so, I couldn't keep up. And then when I couldn't understand it anymore and I was watching the video clips that they have and none of it made sense, it piled, and piled, and piled. I just felt like there was nothing I could do. There was no one I could talk to to pull out of it, to get help from it. And so, I just stopped going altogether. Stopped doing it all. I'm not going to do it at all…for this semester.

Developmental mathematics courses across the nation have been characterized by high enrollment numbers and withdrawal rates (Bonham & Boylan, 2012; Cafarella, 2014). Not only have those effects been costly for state-funded institutions (Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014; Chen & Simone, 2016), but research has indicated that overall student probability of degree completion decreases as time in developmental sequences increases (Bailey, 2009). Research often has focused on curricular and preparatory reasons for attrition (Bonham & Boylan, 2012; Higbee, Arendale, & Lundell, 2005) leaving the human experience overlooked at the micro-cultural level. In this research study, I will attempt to contribute to the gap in the literature by studying the lived experiences of eight students who withdrew from a technology-based, Intermediate Algebra course at a small, public university in the southwest United States.

This study is educationally significant in providing insight about students who have withdrawn from a developmental mathematics course and who are at risk of not completing their post-secondary degrees. To progress to college-level mathematics, developmental mathematics students were required to pass an intermediate algebra course that, according to the institution's research department, had the school's highest attrition and failure rates. This study occurred at a time when at the national level a large number of students were placing into remediation mathematics courses (Bonham & Boylan, 2012; Boylan, 2011; Cafarella, 2014). The purpose of this phenomenological study was to explore, through personal interviews and Colaizzi's (1978) method of analysis, the lived experiences of students who withdrew from a developmental mathematics course at a small, southwestern university.

Review of the Literature

Over half of the students entering 2-year colleges, and over one-fourth of all undergraduate students in the United States require at least one developmental course to gain necessary skills for placement and/or success at the college-level (Bautsch, 2013). According to the U.S. Department of Education, developmental courses are offered at 75% of colleges and universities to accommodate the growing need of academic remediation (Boylan, 2002; Howell, 2011; Melguizo, Bos, & Prather, 2011). Compared to those requiring developmental English or writing, enrollment in developmental mathematics has been particularly high (Bahr, 2013; Bailey, Jeong, & Cho, 2010; Silverman, & Seidman, 2011). Additionally, Bahr (2010) posited, upon first-attempt failure of a developmental mathematics course, students were either more likely to cease further attempts or experience greater rates of failing subsequent first-attempts of courses within the required developmental mathematics sequence. Researchers have attributed high failure and attrition rates of developmental students to several factors, including poor affect and self-efficacy; challenging course rigor; a lack of basic mathematics facts and study skills; and instructional practices (Bonham, & Boylan, 2012; Boylan, 2002; Boylan, 2011; Cafarella, 2014). Course redesigns, which have often included the use of technology, are being implemented across the nation to address these barriers and increase developmental student success rates (Bonham & Boylan, 2012; Boylan, 2011; Boylan & Saxon, 2012; Cafarella, 2014; Melguizo, et al. 2011). Based on a review of the literature, the number of studies addressing these topics has been increasing over recent decades; however, qualitative developmental mathematics research remains minimal (Higbee, et al, 2005; Koch, 2012).

In a search for studies similar to this research, one quantitative and six qualitative references were selected for review. Five of the study results were published in scholarly journals and two were published in dissertations; all were written since 2007. No article was found that shared all five major attributes of this research: a phenomenological study about students who withdrew from a technology-based, developmental mathematics course. Nonetheless, every article had at least one common element with the study, and most had two or more common elements. According to Higbee, et al. (2005), the lack of qualitative research in developmental education is not a surprise. In fact, they emphasized the underrepresentation of qualitative studies and implored researchers to portray the perceptions and voices of the complex and richly layered developmental student experiences. Higbee et al. (2005) specifically advocated, “Future research that connects the characteristic of students in developmental programs with research on access and retention will help improve teaching and learning in community colleges” (p. 8).

Studies Reviewed

Among the six qualitative studies, interactions with teachers, isolation, and self-efficacy were the most common themes followed by preparedness, technology barriers, and reflections. The most relevant article was a phenomenological study of 13 students from a private, 4-year institution who did not pass a developmental mathematics course (Cordes, 2014). Several students had taken one or more of the classes multiple times. Through formal response questions, interviews, and a self-efficacy questionnaire, Cordes (2014) was able to identify 10 emerging themes: “(a) isolation, (b) self-doubt and negative attitudes towards developmental math, (c) success clouded by inability to progress, (d) fixed mindset, (e) experiences with teachers, (f) expected placement, (g) good placement, (h) desire for change, (i) overall positive experience with staff, and (j) change in math confidence” (p. 162). The instructional-delivery model for the mathematics classes was the emporium model, where students did their work on a computer in a lab setting. The lab was populated with staff, tutors, and students, while formal instruction was delivered once a week by the instructor in a traditional lecture format. Cordes (2014) commented on the need for instructors to understand their students’ affective characteristics and personal experiences to improve skills, persistence, and mindset growth.

Two other studies specifically focused on students who were or had enrolled in developmental education courses (Barbatis, 2010; Koch, Slate, & Moore, 2012). Koch et al., (2012) used a phenomenological approach to explore students' perceptions of developmental courses as experienced by two developmental mathematics students and one developmental writing student at a community college in Texas. Coding the interview transcriptions unveiled five emerging themes: affective perceptions, academic perceptions, behaviors, resources, and perceived benefits. Koch et al. (2012) highlighted the misalignment of students' initial expectations with the stark reality of rigor and time commitment needed to pass the courses. Additional emphases in the study were placed on the influences of teacher behavior, both positive and negative, as well as the importance of time management skills, goal identification, technological barriers, overcrowded classrooms, and self-efficacy.

In another study, Barbatis (2010) addressed factors contributing to persistence. Results of 18 students who were enrolled in and/or graduated from a community college were compared with four students who entirely withdrew from school (Barbatis, 2010). Enrolled students had earned at least 30-credit hours. All but two students were members of ethnically minority groups, and all participants had been required to take the three developmental courses offered at the institution (i.e., reading, English, and mathematics) as well as participate in a first-year experience learning community. Key factors in retention included student-faculty interactions, preparedness, and use of resources (Barbatis, 2010).

Bambara, Harbour, Davies, and Athey (2009) conducted interviews of 13 community college students enrolled in high-risk online courses in the American

Southeast to examine their lived experiences. The courses were defined as having at least 30% failure or withdraw rates. The emerging themes "were isolation, academic challenge, ownership, and acquiescence" (p. 223). Withdrawal decisions among participants were imminent when tangible rewards were no longer a result of their investment. Other contributing factors included the lack of faculty-student relationships and difficulties with technology. In a similar vein, online graduate nursing students reported the same two primary reasons for withdrawing from their program, along with personal reasons (Perry, Boman, Care, Edwards, & Park, 2008). The study was of particular interest due to the critical differences between it and the current study in academic discipline, level of education, and course delivery method, while remaining consistent with perspectives about withdrawing. Upon prematurely exiting the program, students were required to send an email to the school's administration identifying their reasons for withdrawal. Perry et al. (2008) learned an additional key message from the emails: the decision to withdraw was not taken lightly but rather seemed to be the only plausible solution at the time. Of the 86 students in the study, 31 intended to return upon addressing criteria that informed his or her choice.

In the last of the six qualitative studies reviewed, Minnick (2008) uncovered multiple themes from interviewing six students who withdrew from the University of Montana at the end of their first year. Themes were broken into three major categories: college transition and adjustment, the decision to drop out, and looking toward the future. The story that was painted by these students began with excitement and hope that turned to confusion and a sense of overwhelm. Students first became unfocused and then felt isolated and unmotivated followed by disappointment and embarrassment. Struggling with their decisions to persist, they finally succumbed to leaving and began making new plans.

The one quantitative study reviewed was conducted in 2008 at an urban community college in Texas, where data from 9,200 first-time-in-college students were analyzed over a 4-year period to determine predictors of retention (Fike & Fike, 2008). Using point-biserial correlation coefficients, the researcher discovered lower retention rates for students who did not enroll in developmental mathematics compared to those who did enroll in developmental mathematics, even if students in the enrolled group did not pass.

The literature also addressed specific characteristics common to developmental mathematics students to include being underprepared, having poor study skills, and experiencing low self-efficacy (Boylan, 2002; Hagedorn, Siadat, Fogel, Nora, & Pascarella, 1999; Zientek, Fong, & Phelps, 2017). Stigler, Givvin, and Thompson (2010) surveyed 748 developmental mathematics students and found that students' procedural knowledge was more prevalent than conceptual knowledge, and reasoning skills were rarely required or developed in prior experiences. Stigler et al. (2010) concluded that future redesigns should be informed by developmental mathematics students' prior knowledge and experiences.

Theoretical Framework

Symbolic interactionism is a social theory developed in the 1920s by sociologist George Herbert Mead. While Mead was credited for defining the theory, his student Herbert Blumer further developed the framework following Mead's death in 1931. Blumer was also credited for naming the theory *symbolic interactionism*. Symbolic interactionism frequently has been used in the literature to gain understanding of human perception in social engagements and was based on three main tenets (see Figure 4.1): meaning causes action, meaning can be different, and meaning can change. Humans perceive reality and act based on personal interpretations of words or symbols, to include objects, people's perceptions of a social encounter can be different while equally correct, and people's perceptions can change (Blumer, 1986).

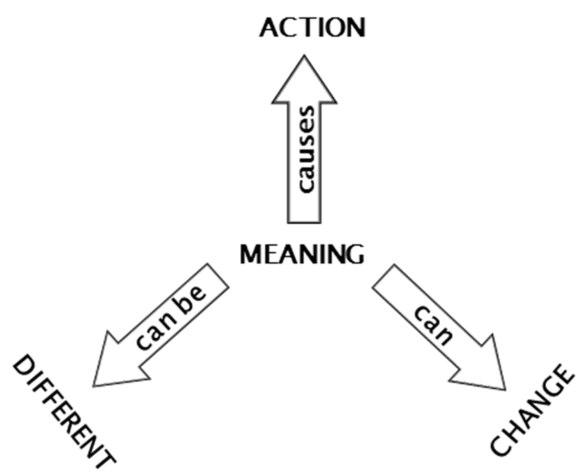

Figure 4.1. Symbolic Interactionism Diagram based on Blumer (1986).

While the symbolic interactionism theory can be useful in qualitative research, its subjective nature often has been a deterrent in quantitative studies, as it is difficult to measure for hypotheses testing or generalizability (Blumer, 1986; Charon, 2009; Willis, Jost, & Nilakanta, 2007). Nonetheless, phenomenological researchers, as well as other qualitative interests in sociology, anthropology, psychology, and education, resonate with the flexibility and humanistic qualities of the theory. The interactionist's ability to abandon universal truths is the very skill that reveals rich meaning within specific sub-cultures (Willis et al., 2007).

From post-structural and postmodern perspectives, the plurality of interpretations present in social interactions adds to our understanding of humans, cultures, and the world around us. This research will advance the theory of symbolic interactionism by contributing to the pool of information gathered about individual perceptions within social contexts.

Social interactionism in mathematics. The process of teaching and/or learning mathematics relies heavily upon human interaction and, therefore, frequently has been framed in the literature by symbolic interactionism. Whether the focus is on the teacher, student, pre-service educator, administration, facility, or socio-cultural atmosphere depends upon the interest of the researcher. While many mathematical studies have employed symbolic interactionism as a theoretical launching pad for identifying objects, actors, and interpretations of actions (Bonner, 2014; Brandt, 2013; Font, Godino, & Gallardo, 2013; Lutovac & Kaasila, 2011), others have used it more as a taxonomy to categorize and structure the research questions and scope of analyses (Bouchey, 2004; Teo & Osborne, 2012). A third use combines symbolic interactionism with one or two other theories to explore specific perspectives of the learning process (Krummheuer, 2013; Rasmussen, Wawro, & Zandieh, 2015). Regardless of the modality, the theory is not uncommon to research in mathematics education and can contribute to a deeper understanding of human behavior in the mathematics classroom.

As a launching pad, Lutovac and Kaasila (2011) used symbolic interactionism to express a pre-service teacher's perception of her mathematics identity. Based on those markers, narrative rehabilitation and bibliotherapy techniques were developed to aid participants in recognizing, processing, and changing their beliefs to better equip them as future mathematics teachers. In a similar vein, Brandt (2013) used symbolic interactionism as a basis for describing learning outcomes of kindergarten students as related to teachers' pedagogical interpretations of children as learners. Bonner (2014), who used grounded theory to study the characteristics of three successful mathematics teachers of underserved students, constructed theoretical implications of culturally motivated interactions from themes inherent to symbolic interactionism.

Categorical uses of symbolic interactionism can be explored through the research of Bouchey (2004), as well as Teo and Osborne (2012). Bouchey (2004) investigated mathematics student perceptions whereas Teo and Osborne (2012) considered implications of curricular change at a STEM preparatory school. While Bouchey (2004) expanded upon a single tenet of symbolic interactionism – reflected appraisals, Teo and Osborne (2012) highlighted five tenets, which were

> (a) face-to-face social encounters entail interactions between people, (b) social interactions are a process forming human conduct or behavior, (c) human subjects are agentive actors in the creation and interpretation of meaning, (d) the ''objects'' of analysis can be indicated, pointed to, or referred to, and (e) actions are interlinked. (p. 544)

Using symbolic interactionism in combination with other theories also has been prevalent in the literature. Mathematical instruction in a Linear Algebra course was the topic of interest for Rasmussen et al. (2015), who developed an interpretive framework consisting of both cognitive and sociocultural perspectives to capture four constructs:

disciplinary, participation, mathematical conception, and classroom practices. Krummheuer (2013) found co-frameworks advantageous as well in his grounded theory research on mathematics learning in early childhood education. In this case, symbolic interactionism was combined with constructivism to better understand the processes of diagrammatic and narrative argumentation in mathematics learning concepts.

Symbolic interactionism can provide a unique theoretical framework to explore human interactions and subsequent interpretations and actions in the field of mathematics education. It can be used as a basis of reasoning, a taxonomy of investigative sub-topics, or in conjunction with alternative theories to probe deeper into the human experience of the mathematics learning model. Symbolic interactionism's comprehensive yet flexible essence renders it a strong framework in the quest for understanding the experiences of students who have withdrawn from a technology-based, Intermediate Algebra course.

Method

Participants

Due to the nature of information desired, phenomenological studies benefit from purposeful sampling strategies (Coyne, 1997; Creswell, 2013; Merriam, 2014). The goal of this type of research is to identify specific groups or individuals who may shed light on a topic of interest and, instead of using the results for generalizations, find richness and depth in the details (Merriam, 2014). Criterion sampling, a purposeful strategy, was used based on an interest in participants who shared the common experience of having withdrawn from the Intermediate Algebra course (Creswell, 2013). For some developmental mathematics students, achieving success was hindered by situational reasons (see Zientek, Schneider, & Onwuegbuzie, 2014). Situational factors are not specific to mathematics and might influence withdrawals or success in multiple courses. Because the focus of this study was on withdrawals from Intermediate Algebra and not withdrawals from college in general, students selected for interviews had withdrawn from Intermediate Algebra in a previous semester but did not withdraw entirely from school.

The study was conducted in Intermediate Algebra course sections at a small, open-enrollment, 4-year university in the southwestern United States. Multiple sections of Intermediate Algebra were offered in the Fall 2017 semester. Intermediate Algebra was the higher of two developmental, pre-requisite mathematics classes for students seeking admittance to college-level mathematics. Students were admitted into Intermediate Algebra based on earning a C or higher in the previous developmental mathematics course or by an appropriate score on a placement test.

The institution once was a community college and still serves as such along with new university responsibilities. The institution consisted mostly of 18-24-year-old, White students followed by Hispanic, Unknown, African American and Native American. More females (55.8%) attended compared to males (44.2%). In the Fall 2017 semester, Intermediate Algebra students consisted mostly of freshmen (83.3%), followed by sophomores (9.2%), juniors (4.0%), and seniors (2.0%). The institutional pass rate for Intermediate Algebra was 52.2% while the attrition rate was 22.8%.

To identify participants, a survey was offered as an extra-credit assignment to all students enrolled in Intermediate Algebra during the Fall 2017 semester. The survey asked whether the student previously had withdrawn from an Intermediate Algebra course at the hosting institution and, if so, would the student be willing to participate in an interview about their experience. Three students who met the criteria responded and were interviewed. Five additional students were identified through institutional records as having met the criteria and, after giving consent, were interviewed as well. A total of eight participants were interviewed. Seven of the participants were male and one was female, ranging in age from 20-57 years. All eight participants were White. A more demographically diverse group was desired but based on criteria and consent; these were the only students available for the study. All participants previously had withdrawn from a technology-based Intermediate Algebra course at this institution.

Five of the eight interviewees had grown up in the local area. Two participants were first-year students, both seeking a degree in Graphic Design. None of the other students shared a chosen academic major. Four return students were among the sample, each with a family and/or career. Five students passed the Intermediate Algebra course shortly after the interviews were conducted and were able to advance to their perspective college-level mathematics courses.

Research Design

An interpretivist approach was used for this phenomenological study to gain the essence of the shared, lived experiences of students who chose to withdraw from Intermediate Algebra (Maxwell, 2005; Moustakas, 1994). This approach seemed best based on multiple individuals having a similar experience (Creswell, 2013). In addition, this approach was consistent with my preference of the constructivist-interpretivist paradigm, which relies on the interaction I have with each participant to stimulate hidden realities of their personal encounters (Ponteratto, 2005).

Although my perspective is nomothetic in nature, based on a desire to look for a more systemic explanation of the phenomena, I still hold to a constructivist-interpretivist philosophical paradigm, which allows individual truths to be different, yet equally correct. My hope, among the multiple realities, was to discover some emic constructs that were consistent among the socio-culture of students who have withdrawn from developmental mathematics courses. I subscribed to an axiological parameter of research because I did not think it possible to divorce my beliefs entirely from the reality of the participants. However, I can bracket them and disclose them to the reader to provide distinction and trustworthiness (Ponterotto, 2005).

Author's autobiographical statement. As a young adult, I dropped out of four different institutions before I finally completed a semester and progressed toward the completion of my bachelor's degree. I often wondered if my teachers thought about my withdraw decisions in a similar context as I have thought about students who withdrew from my classes. I have been teaching developmental and college mathematics for 19-years and have witnessed the attrition of hundreds of students. In qualitative research, it is important to bracket the researcher's history, perceptions, biases, and assumptions to create a more authentic accounting of the story being told (Creswell & Poth, 2018).

My personal choice to repeatedly withdraw from school derived from a genuine disinterest in academia combined with a lack of desire to perform, meet deadlines, and/or adhere to the institutional structure of learning. I realize my students' reasons often differed from mine, which cultivated a keen interest in expanding my understanding of the phenomenon. Specifically, I became interested in learning about the experiences students had that ultimately contributed to their decisions to withdraw from Intermediate Algebra at my institution.

I expected to learn about personal and socio-economic reasons, as well as a few decisions related to improper course placement. However, my deeper interests included characteristics of the Intermediate Algebra experience that served as a deterrent to retention, such as, the difficulty of the material, the pace of the course, the role of the instructors, the physical environments, the instructional delivery systems, and other factors that could be addressed by either the institution or the instructor to improve retention. Based on a pilot study I conducted, I chose interview questions to invite multiple perspectives from people who shared this experience and hoped to raise the voices from often-silent students who changed their minds about completing Intermediate Algebra, at least for one semester. As an advocate for multiplication-fact automaticity, I must admit I believe the acquisition of mathematics concepts adheres to Bloom's taxonomy regarding the cognitive process dimension (Krathwohl, 2002). Specifically, to be successful in an Intermediate Algebra course a student should have a mastery level of multiplication facts. Further, I believe that students with weak multiplication-fact automaticity experience a cognitive overload with the complexity of the knowledge for which they find themselves ill prepared. Finally, I believe that student weaknesses in multiplication-fact skills are a contributing factor to perceptions that lead to withdrawal decisions. For these reasons, I have included questions about the participants' self-efficacy concerning his or her positive and negative experiences, multiplication-fact automaticity, memories of formal multiplication-fact instruction, and calculator dependency. However, as an outside observer, I attempted to bracket my biases and act as a neutral instrument of data collection while delighting in the gained insight. I subscribe to authentic research practices and was mindful of my role as a researcher to remain investigative about things I do not fully understand and quiet about my preconceived notions and biases.

Procedure

Upon IRB approval, an electronic demographic questionnaire was administered to consenting Intermediate Algebra students during the Fall 2017 semester. The document included a question about whether the students previously had withdrawn from the Intermediate Algebra course. The interviews of the eight participants began with a verbal explanation of the purpose and method of the research. The interviewer/researcher attempted some informal conversation for relaxation purposes and audio-recorded the interviews as outlined without taking notes. Interviews were transcribed in preparation for analyzing the data.

Instrumentation

The instrumentation consisted of the researcher/interviewer, the interview protocol (see Appendix C), the interviews, and a reflexive journal. The interview protocol was developed to represent the research questions, while remaining grounded in both the literature review and the conceptual framework that served as beacons for the study. Spradley's (1979) protocol guided the research using "grand tour, mini tour, example, experience, and native language questions" (p. 86), while considering Kvale's (1983) examples of introducing- and probing-type interview questions. Semi-structured interviews were conducted. Questions were pretested by the researcher with a small sample of participants who withdrew from the same course in different semesters.

Data Analysis

Consistent with the social-interpretivist approach, as outlined by Moustakas (1994), several rounds of coding were conducted. A specific coding technique mentioned by Moustakas (1994) is that of Colaizzi (1978). This approach uses a general guideline for analyzing and coding interviews. The researcher transcribes the interviews and reads through them several times to get a general idea of the experiences of the participants. The researcher begins coding by reading the transcripts again and identifying significant statements. The statements were compiled, and the researcher assigns formulated meanings to each statement. The formulated meanings can be expanded or collapsed to reduce repetition and create clusters of information. The clusters were also expanded or collapsed to accommodate emerging themes representative of all participants. Upon finalizing themes, a full-scale description was produced of the phenomenon using frequent in vivo coding (Creswell, 2013) to preserve the language of the participants. Examples of significant statements, formulated meanings, and themes are also presented in table format (Creswell & Poth, 2018). For this study, the researcher included interpretive notes and memos along the margins of the transcriptions, as well as in a separate notebook, to clarify ideas, add background knowledge, and reflect.

Data were collected through interviews, observations, and field notes. To increase the credibility and trustworthiness of the study, four methods of validation were employed: member checking, multiple forms of data collection, reflexive journals, and peer-reviewing (Creswell, 2013; Maxwell, 2005; Merriam, 1988). Member checks were conducted by sending transcripts to the participants for comments and correction. A focus group was also conducted with a subgroup of the participants to review the exhaustive description of the phenomenon. Relevant input informed necessary changes. A personal reflexive journal was used by the researcher as an audit trail. Finally, peer-reviewing provided insight to the interview protocol, research questions, and thematic coding.

Results

Eight interviews were transcribed verbatim resulting in 123 significant statements with formulated meanings (see Table 4.1). These statements were grouped into larger clusters, referred to as meaning units, which were regrouped and reduced into six themes. Table 4.2 shows three of the six themes, each with examples of significant statements.

Table 4.1

Some Significant Statements with Formulated Meanings of Students Who Withdrew from a Technology-Based Intermediate Algebra Course

Table 4.1

Some Significant Statements with Formulated Meanings of Students Who Withdrew from a Technology-Based Intermediate Algebra Course

Significant Statements	Formulated Meanings
a lot of the people I know and associate myself with have taken it	beliefs about developmental mathematics
about a week into it my health started failing again. And so that was the only reason why I withdrew	resolve to withdraw
after I was so far behind it was impossible for me to catch up	resolve to withdraw
add the numbers that I would have to get to the answer that I needed rather than memorizing	multiplication facts

(continued)

Significant Statements	Formulated Meanings
because it was cramped, a lot of people were hesitant to ask questions or to say something	external barriers

Significant Statements	Formulated Meanings
and so in a year, I'll be heading up to (another school) to get my major in film and animation	students often have purpose that keeps them in school even if they have to do difficult things
college would still be a good investment in myself even if I didn't graduate or even end up using my degree or my major	students often have purpose that keeps them in school even if they have to do difficult things
dividing is a little bit more difficult	calculator uses

Table 4.2

Some Themes with Significant Statements

Student Goals *Educational goals, professional goals, personal goals*	False Course-Expectations *What was different than their expectation? What new ideas were they faced with?*	The Decision to Withdraw *What prompted their decision to take action and withdraw?*
trying to go to school and make more money	because it was cramped, a lot of people were hesitant to ask questions or to say something	I had five other classes
I decided to go back at school rather than just sitting, programming machines all day	having a test question come out wrong and I wrote it down on paper correctly	my niece needs a heart transplant
I might as well do something productive	getting everyone signed in on mymathlab	I was so far behind

Theme 1: Student Goals

Students were excited about discussing their goals and dreams. Most were working toward a specific degree including Criminal Justice, Business Administration, Graphic Design, Dental Hygiene, Political Science, and Psychology to pursue a career representative of their passions. Many were attending school to eventually "make more money," or start their own business. Others were satisfying personal goals to better themselves, "set a good example," "expand my knowledge," or "become the first person in my immediate family to go to college." Some just enjoyed learning and believed "college would still be a good investment in myself even if I didn't graduate or even end up using my degree or my major."

Theme 2: False Course-Expectations

Upon beginning the Intermediate Algebra class, students described feeling confused, overwhelmed, surprised, and a sense of disbelief. Several factors contributed to these descriptions to include the demands of the course, teacher influences, and a general lack of preparedness. One student recalled he was overwhelmed with "getting into that whole experience."

The course itself was rigorous and fast-paced, which is evidenced by students' responses about the course. One student said she "would spend hours and hours and hours" on the course. Another said, "now you have to remember all these concepts." A third remarked, "the schedules that they have [us] on are pretty quick, so I don't feel like I get enough time" to learn the material. Six participants were enrolled in sections taught using a flipped model, which required students to watch a video lecture prior to coming to class, then working homework problems in class on a computer-based delivery system called MyMathLab. The remaining two participants took online sections of Intermediate Algebra, also delivered through MyMathLab but without the face-to-face interaction of the flipped models. The technology itself was described as confusing "as to what needed to be accomplished on that program for that section that we were going to go through," and created a daily challenge "getting everyone signed in on MyMathLab." Additionally, MyMathLab required students to learn new skills typing mathematical responses and was intolerant of errors or dropping a symbol. It was discouraging "having a test question come out wrong and I wrote it down on paper correctly" but left something out when entering it into the computer.

Relationships with teachers were described as "quick and straight to the point," and in some cases, a student noted, "He [the teacher] had no idea that I was even in the class." One student admitted not knowing how to talk to the teacher, and another said, "Because it was cramped, a lot of people were hesitant to ask questions or to say something." Whether the class was too crowded or purely online, students described feeling a sense of isolation. Even with relatable teachers, students reported the relationship "just never really got established" or was not something "where you can talk person to person."

A general lack of preparedness was commonly expressed during the interviews. A student commented on the course being, "a little bit out of my depth," and chose to take the preceding mathematics course instead. Others said, "I wasn't as prepared as I thought I was," and "it was my first semester, so I didn't understand the tests had to be taken in the testing center." While students did not notice a prevalent outward stigma of developmental mathematics at the school, many grappled with their inner sense of embarrassment and the reality of the course being, “beneath everything else,” and “not even on the college level.”

Theme 3: The Decision to Withdraw

It was difficult to clearly distinguish what prompted each person’s choice to withdraw from the class because choices are often the culmination of many influences. However, each student expressed a moment of clarity in his or her decision that would not be swayed. In fact, the common sentiment was one of no choice at all. Withdrawing was the only viable solution as illustrated through comments like, “this is not happening. My fight or flight took into effect and I needed to get another course,” and, “I just decided to use my time wisely elsewhere.” Other statements included, “I just stopped going altogether,” and “after I was so far behind it was impossible for me to catch up.” The definitiveness of the statements was profound, without reservation, although many students wished a different choice would present itself. Three such examples are, “there's no way I'm going to pass anyway so I'm out of here,” as well as “I even emailed the instructor and he never emailed me back so I guess I'll just take it next semester,” and “I'm not proud of that you know, I probably should have hung in there, but at the time it wasn't an option.”

The previously described withdraw reasons were either academic or a function of time. However, personal situations were also considered in students’ decisions to withdraw, as were persuasive acts from teachers. As one student accounted, “about a week into it my health started failing again. And so that was the only reason why I withdrew.” For another, a family member’s need for a heart transplant, coupled with the demands of the course, proved too big a load. The “drill sergeant” attitude of a teacher was enough to solidify one student's decision, while the "warning" speeches of instructors for two other students stating, "If you're not getting this right off the bat you probably should back out," and, "You're kind of out of luck," weighed heavily in their final decision-making.

Theme 4: Mathematics Experiences

Students were asked to recount both a positive and negative mathematics experience. In all cases, when responding to the positive experience question, students included negative experiences as interwoven parts of the memories. In contrast, this rarely happened when asked about a negative experience: limited or no positive experiences were included in the fabrics of these accounts.

A common thread in negative mathematics experiences included a lack of understanding, such as, "This goes back to that not getting learning of it to begin with. I just don't like the process of how it's taught," and "Just not understanding in fourth or fifth grade. That's when we're heading into division and stuff." The next student got caught up in a state-wide mathematics curricular change and said, "One term it would be one subject of math and none of it would connect between terms which was really frustrating because it's like, ok, forget all of that, now we're doing this." Another student expressed a sense of isolation as well, because the rest of the class seemed to be understanding. The student stated:

> I feel like I was passed by and I needed extra help and everyone else was going forward and I was stuck, and it just made it so every time there was a math class I felt like I was barely getting it or just getting a grade just to pass by.

Positive experiences also reflected a connection to understanding, as in these four examples: (a) “I'm not the best in math, but it feels good to understand it;” (b) “It's always a feeling of accomplishment when you finally get something that you didn't get before;” (c) “Where I feel like it's been explained enough where I feel like I can do it;” and finally, (d) “Part of me likes math in a way. I like solving problems. There's sort of a satisfaction that comes when you do solve it.”

A sense of application or practicality rendered an experience positive as well. However, one might argue a level of understanding would be implied. Two examples are, “So I think learning math and applying it to something was pretty fun,” and “I can actually help my little sister now with math.”

A third reason for perceiving a mathematical experience as positive had everything to do with a helpful teacher. One student commented, “My first semester at my new high school was really tough but I had a great math teacher. She worked with me every day after school to make sure I passed the class.” Another student said, “My fifth-grade teacher, he was very wonderful, and he would have these root beer barrels that he would always give out and he was the only math teacher I really remember trying to really help.” Finally, “He's a good guy. I think he's dedicated, and he's given a lot of his time to the classroom and to myself, especially. I've needed some extra help and some direction.”

Theme 5: Strategies for Success

Every student formulated a new plan to increase their chances of success next time they took Intermediate Algebra. Most plans addressed factors that led to their decisions to withdraw. For instance, the student with the health issue decided to return once his condition stabilized. Students who initially carried heavy course loads reduced them to allow for more time towards mathematics. Some students chose to complete the bulk of their non-mathematics courses, so they would be more comfortable with the college experience, school schedules, and the facilities before tackling the complexities of mathematics and technology. Others chose a different teacher; different format; and/or decided to utilize the many forms of instructional resources and assistance available at the institution. One quote sums the concept up well: “Know what you're getting into, go to the math tutoring center all the time, and then don't be afraid to ask questions.”

Theme 6: Mathematics Self-Efficacy

Students were asked if they had memorized their multiplication-facts and when they recalled learning them. One student confessed he had only recently worked on them and still used the counting-up method for his 6’s, 7’s, and 8’s. The rest of the students identified elementary school as the time and place where they initially learned their facts, but only three students reported knowing the 0’s through 9’s by memory. Their accounts were consistent with their responses to the question about calculator usage. Those who reported having their multiplication-facts memorized responded to the reason for using a calculator question with, “If I know it’s not realistic to think I can do it without the calculator,” and “If it’s a really complex one.” They made the distinction of using the calculator for speed, accuracy, and complex problems outside of typically memorized facts. The following quote illustrates this point.

> I try to use my head way more than my calculator. If it’s a bigger number, I’ll check to see if it has a perfect root. Or if I’m multiplying huge numbers like 57 times 93, or for long-division problems.

Students who did not have all 0-9 integer multiplication-facts memorized typically identified 6's, 7's, and 8's as "kind of wild" or "the hard ones." They relied on the counting-up method, where they would, "add the numbers that I would have to get to the answer that I needed rather than memorizing." Their responses to the calculator usage question included, "I'll pretty much use it for what I don't remember," and "totally dependent, like 100%," and "What two numbers make 24, then what two numbers make 28, then 26, so I'm like, was that 7 times 4 or was that 4 times – you know what I mean?" They also expressed a need to use the calculator for dividing when working with radicands and for integer signs.

A contrast emerged in mathematics self-efficacy responses between students who reported they had memorized multiplication-facts and those who did not. Those who memorized their multiplication-facts did not refer to poor mathematical abilities rather they reported neutral or positive experiences, such as "I was just kind of in the middle of the pack and it was a comfortable place to be," or, "I always enjoyed math". In one case, the student reported nothing at all. Those who did not claim to have their multiplication-facts memorized reported either negative beliefs about their mathematical abilities, "I'm terrible with numbers," and "Every time there was a math class I felt like I was barely getting it," and "throughout high school I just struggled with math and I always have," or negative experiences overall, such as, "I hated math all through high school and prior to high school" and "high school [math] was really tough."

Discussion

In this study, eight students shared their experiences of withdrawing from an Intermediate Algebra class. Initially, motivation to successfully complete the course and progress toward degree completion was the consensus; to be replaced with feeling overwhelmed and surprised when their expectations did not align with the realities of the course. They scrambled to adjust and most worked many hours on coursework to try to keep up. Ultimately, circumstances overcame their abilities to succeed and they surrendered to recover, regroup, and return. Strategies were sought to influence future attempts and improve the likelihood of success amid skepticism of self-concept, abilities, and future plans. They all showed up the following semester, and five of the eight succeeded!

Five of the 10 topics that emerged in Cordes' (2014) study about experiences and perceptions of developmental mathematics students were particularly similar with the topics identified in the current research: isolation; inability to progress; experiences with teachers; negative attitudes; and a desire for change. Theme 2 (False Course-Expectations) of this research exposed the sense of isolation and an inability to progress based on the demands of the course; a belief of no available help; and the inability to access teachers as a viable resource. In Theme 4 (Mathematics Experiences), connections were drawn between students' experiences in the Intermediate Algebra course, and negative attitudes cultivated by mathematical memories. Both themes were represented in the research of Koch, et al. (2012) and could be tied to vicarious and mastery experiences (see Bandura, 1997). Theme 5 (Strategies for Success) was a new expression of hope, powered by personal change, and overlapped with the discoveries of Perry, et al., (2008). Theme 1 (Student Goals) spoke of the excitement and hope of reaching goals while Theme 3 (The Decision to Withdraw) was the result of misconceptions and overwhelming circumstances. In the study by Minnick (2008), similar themes emerged when six freshmen from the University of Montana spoke of their initial arrival at school, followed by unexpected circumstances, and the choice to withdraw.

Of particular interest to me as a mathematics teacher were the responses from Theme 6 (Mathematics Self-Efficacy), which described students' perceptions of their multiplication-facts and calculator usage. Their answers revealed a relationship between multiplication-fact skills, calculator dependency, and self-efficacy, as understood through their described mathematical experiences in Theme 4 (Mathematics Experiences). Because self-efficacy has been identified as one of the best predictors of student success, a discussion of self-efficacy is warranted.

Across multiple studies, mathematics self-efficacy tends to be the best predictor among various factors known to predict student success (see Zientek & Thompson, 2010). Self-efficacy has been defined as "the beliefs students hold about their academic capabilities" (Usher & Pajares, 2009, p. 89). Mastery experiences and emotional and physiological states are two of four sources of self-efficacy (Bandura, 1997; Usher & Pajares, 2009). Research suggests mastery experiences is a source of self-efficacy that has been identified as more powerful than the other three identified sources (Zientek et al., 2017) and are acquired through prior successes and failures. In mathematics, physiological states often refer to mathematics anxiety, which has tended to be exhibited at higher levels in developmental mathematics students than the general population (see Zientek, Yetkiner, & Thompson, 2010). Vicarious experiences and social persuasions are the other two sources of self-efficacy (Bandura, 1997) and have been identified as predictors of developmental mathematics students' skills (Zientek et al., 2017).

The specific calculator functions upon which the students with low multiplication-fact automaticity felt a dependency were fractions, division, radicands, signs, and finding factors. Excluding operations with signs, these mathematical computations are heavily dependent on multiplication facts. Further, research has indicated that multiplication-fact automaticity is a predictor of students' abilities to perform fraction procedures (Hansen et al., 2015) and fraction procedures have been advocated as a predictor of algebra abilities (U. S. Department of Education, 2013; Zientek, Younes, Nimon, Mittag, & Taylor, 2013).

As a final observation, Theme 5 (Strategies for Success), which spoke of implementing change to increase chances of success upon retaking the course, was laden with non-mathematical skills-building strategies. As thorough as the participants were in describing what they learned from their mistakes in the first attempt, it was surprising to discover the void in their abilities to identify specific computational skills that, upon practicing, could contribute to their follow-on success. The lack of preparedness, as it relates to basic skills, coincides with the research of both Barbatis (2010) and Bambara, et al. (2009).

In Conclusion

The purpose of this research was to understand the lived experiences of eight students who withdrew from a technology-based, Intermediate Algebra course. Symbolic interactionism provided a lens that enabled the researcher to consider several actors in the model that may or may not have contributed to the students' perceptions and ultimate decisions to withdraw. Such actors included the teachers; time; technology; the physical classroom spaces; students' individual histories of mathematics and learning; cultural predispositions; relevance; workloads; a personal account of multiplication-fact automaticity and calculator skills; health; and familial and professional responsibilities. Through symbolic interactionism, the researcher can explore how these actors, or variables, prompt action, differ among individuals, and perhaps inform change.

The culmination of these lived stories portrayed the essence of the experience of having withdrawn from Intermediate Algebra at this institution. These students had an earnest desire to successfully complete the course in pursuit of their personal, professional, and educational goals only to be met with a complete misunderstanding of the course demands and expectations. For most of them, hard work was met with "confusion," "frustration," a sense of "isolation," and not knowing how to access help. For those with low mathematic self-efficacy, these feelings were all too familiar as they compared them to memories of pain and hopelessness, save a fleeting moment or two of clarity or relevance, or perhaps a teacher who was able to bring joy to a student's life amid mathematical goings-on. The choice of withdrawing was no choice at all as many looked for alternatives. Alas, the decisions were made with regret for some; for others "embarrassment," and defeat. At the dawn of their new plan, all of them shifted their mindsets to prepare for next time. They would "reduce their loads," take care of the medical issue, "buy the textbook," "go to the tutoring lab," "switch teachers," choose a different instructional format, "ask questions," and most of all, not be surprised! None of the students discussed how they would address existing deficits in basic mathematics skills.

This research contributes to existing literature about the disconnect between student preparedness and classroom expectations in developmental mathematics. It further aligns with developmental mathematics student characteristics identified in the literature: low self-efficacy, poor study skills, and lack of conceptual knowledge. The findings support the importance of student knowledge and experiences regarding the development of appropriate course expectations and resources, to include student-teacher relationships, basic fact acquisition, placement procedures, organized instruction, and an emphasis on comprehension. In an effort to increase retention and subsequent degree completion, this research adds to the collective knowledge of developmental mathematics experiences and the impact those experiences may have on student success.

Future Research

This group of participants was limited to students who previously had withdrawn from an Intermediate Algebra course but chose to retake the course. A future qualitative study illuminating the experiences of students who chose not to return would be enlightening and an interesting contrast to the current study. Additionally, future studies that seek to understand student perspectives of self-efficacy in relation to basic skills and mindset would inform preparation and prerequisite practices.

Implications for Developmental Education

Understanding the lived experiences of students who chose to withdraw from an Intermediate Algebra course would add to the gap in literature and contribute to the knowledge of retention in developmental mathematics. Qualitative studies are rich in patterns and insights; are postured to benefit students, educators, and administrators; and are not detectable through statistical analyses and quantitative practices (Higbee, et al, 2005). Understanding the experiences of the participants in this study might prompt change in student-teacher engagement, marketing of resources, student self-efficacy, personalizing basic-skill improvement strategies, and fostering an appreciation for overwhelming demands inherent of some developmental mathematics courses. Although phenomenological studies are not meant to be generalizable, the student voices in this study resonate with current literature highlighting these concerns (Bambara, et al. 2009; Barbatis, 2010; Cordes, 2014; Koch, et al. 2012; Minnick, 2008; Perry, et al. 2008).

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CHAPTER V

AN INTEGRATED CONCLUSION

Educators across the nation are concerned about the high failure and attrition rates in developmental mathematics courses and are seeking ways to increase student success by improving elements such as teaching methodologies, learning environments, instructional venues, and noncognitive skills (Bailey, 2009; Bonham & Boylan, 2012; Boylan, 2011; Boylan & Saxon, 2012; Cafarella, 2014). The purpose of this research was to understand multiplication fact automaticity (MFA) within the domain of Intermediate Algebra and to explore the extent to which automaticity is related to student completion and/or success. High automaticity was defined as having memorized the information well enough to have quick, retrievable access with little cognitive effort and was measured by a score at or above the sample median.

While multiplication of integers is a basic skill embedded within typical Intermediate Algebra course objectives, there is little research identifying the benefits of high multiplication fact automaticity (MFA) over the use of multiplication fact retrieval tools, such as calculators, counting techniques, and/or multiplication charts. In addition, specific studies relating MFA to developmental mathematics students are not common. In this research, three separate studies were conducted, interwoven by the thread of MFA: (a) examining differences of MFA test results among eight comparison variables (the number of semesters lapsed between initial college enrollment and enrollment in a mathematics course, mathematics placement scores, first three-semester test scores, end-of-course grade, and attendance); (b) comparing medians between MFA scores and five Intermediate Algebra unit test questions; and (c) capturing the human experiences of eight students who withdrew from an Intermediate Algebra course.

Study 1

The intent of the first study was to diminish the gap in literature by exploring whether students with high MFA were more successful in their developmental mathematics courses than those with low MFA. The sample of developmental mathematics students tended to score well on the MFA test: the median was .92. Students were categorized as high MFA if their MFA score was at or above the median score and categorized as low MFA if their MFA score was below the median. Even though scientific calculators were permitted on all coursework and course assessments, statistically significant differences existed between those with high and low MFA for each of the unit tests and the end-of-grade score. Students with high MFA scored better. The largest differences in means occurred on unit 3 test, over 12 points, while the smallest difference in means, about 6 points, occurred on unit 2 test.

The results support existing literature that showed MFA to be a predictor of successful fraction operations (Hansen et al., 2015) and reinforces Wurman and Wilson's (2012) claim that "Arithmetic is the foundation. Arithmetic has to be the priority and it has to be done right" (p. 47). These findings suggest that educators can increase success rates in Intermediate Algebra by requiring MFA as a prerequisite. Placement scores, attendance, time-lapse between institutional enrollment and first mathematics course, and withdrawal decisions were not statistically significantly correlated with MFA status.

Unit 1

There was an approximate 9-point spread between those with high and low MFA on the unit 1 test. Approximately 75% of the students with high MFA scored at or above the median score of students with low MFA. A Cohen's d of –.46 means that 67.72% of the high MFA group would be above the unit 1 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 62.75% chance that a person chosen at random from the high MFA group would have a higher unit 1 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012). Four of the questions involved solving equations with fractions, which required employing multiplication-fact knowledge when finding common denominators. Missing those four questions, however, could still result in a score of 75%. The mean score for students with low MFA was 66.88%, which is a curious discrepancy that would be an interesting continuation of the study. Of the remaining 12 questions, seven were story problems requiring students to translate words into mathematical representations before solving. None of those 12 questions, however, was highly dependent on multiplication-fact automaticity. The mean score for students with low MFA was 66.88%.

Unit 2

Unit 2 test had the least amount of questions requiring factoring; four systems of linear equations questions. Factoring is a complex, multiplication-fact dependent mathematics skill. If students chose to use the addition method, a greater knowledge of multiplication facts was needed to determine a common multiple; however, students had the flexibility of using the substitution method, which is not multiplication-fact dependent. Regardless, a student could miss all four of those questions and still earn a score of 75%. The mean score on that test for students with low MFA was 71.99%. A Cohen's d of –.32 means that 62.55% of the high MFA group would be above the unit 2 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977;

Magnusson, 2014) and there was a 58.95% chance that a person chosen at random from the high MFA group would have a higher unit 2 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012).

Unit 3

Unit 3 test had six questions that required factoring. Of the six, four were trinomials and two were binomials. The highest score a student could earn without the ability to factor was 62.5%. The mean unit 3 test score for students with low MFA was 66.06% compare to a mean test score of 78.49% for students with high MFA. Like the unit 1 test, approximately 75% of the students with high MFA scored at or above the median scores of students with low MFA. A Cohen's *d* of –.53 means that 70.19% of the high MFA group would be above the unit 3 test mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 64.61% chance that a person chosen at random from the high MFA group would have a higher unit 3 test score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012). More outliers were present for grades of students with high MFA compared to students with low MFA.

End-of-course grade

The cutoff grade for advancing into a college-level course was 70%. For end-of-course grades, there was a statistically significant difference between students with high and low MFA and the difference in mean scores was 7.5 points. More noteworthy, however, was the fact that the mean for those with high MFA (M = 74.04) was considered a passing score for the course whereas a mean for those with low MFA (M = 66.54) was not considered a passing score. Thus, students with high MFA were more likely to advance to a college-level course than students with low MFA. A Cohen's *d* of –.35, 63 means that 68% of the high MFA group would be above the end-of-course grade mean of the low MFA group as calculated from Cohen's U_3 (see Cohen, 1977; Magnusson, 2014) and there is a 59.77% chance that a person chosen at random from the high MFA group would have a higher end-of-grade score than a person chosen at random from the low MFA group (see Magnusson, 2014; Ruscio & Mullen, 2012).

For each of the unit tests and the end-of-course grades, comparisons of groups with high and low MFA were statistically significant (i.e., $\alpha < .05$) and had practical significance (i.e., effect sizes in medium range). In the case of the end-of-course grade, a 7.5-point difference in means was enough gap to discern whether the students would advance to the college-credit course.

Comments

Literature on Cognitive Load Theory (CLT) highlighted the importance of not overloading working memory (WM) while learning various levels of complex information (see Kalyuga et al., 2011; Sweller, 1994). Additionally, intrinsic load, which has to do with the complexity of the learned material, can be reduced with less interactivity of elements, while automation skills reduce the use of mental faculties on competing stimuli (Plass et al., 2010). Cognitive load was not directly tested in this research, but the tenets of its theory support the findings.

The study was limited to students attending an Intermediate Algebra course during the Fall 2017 semester at a small, public university in the southwest United States. Participation was voluntary. The results are not generalizable but have added to the literature by examining MFA on a sample of developmental mathematics students. The hosting institution will use the insight gleaned from this study. A formal MFA pre-requisite will be recommended for Intermediate Algebra students with a minimum score of 92%. This cut-off score is an illuminating outcome as it challenges generalizations about seemingly good scores less than 92%. For future studies, the exploration of specific relationships between MFA and factoring trinomials is recommended; as well as determining the extent, if any, of predictability MFA has on the comparison variables; and formative assessments conducted by the hosting institution to track outcomes of changes made to the Intermediate Algebra course as a result of this study.

Study 2

The purpose of the second study was to investigate differences of multiplication-fact automaticity between groups of students who either could or could not solve five specific problems selected from unit tests administered in an Intermediate Algebra course: (1) linear equation with fractions, (2) system of linear equations, (3) factor by grouping, (4) simplify a rational expression, and (5) simplify a radical expression. Statistically significant differences existed on multiplication-fact automaticity median scores between groups for problems 3, 4, and 5, and students who correctly answered those problems had median scores at or above 94%. These findings support existing literature linking multiplication-fact retrieval to factoring (De Brauwer & Fias, 2011; LeFevre & Morris, 1999) and suggest the possibility that prerequisite requirements of multiplication-fact automaticity at or above 94% could increase student success rates in Intermediate Algebra. In contrast, no statistically significant differences existed on multiplication-fact automaticity scores for problems 1 and 2. For solving those two problems, students could have used alternative techniques that would not have benefitted from previously having achieved multiplication-fact automaticity.

Problem 3: Factor by Grouping

Instructions for this problem were to factor the polynomial by grouping. Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups (p = .029). The median score for the group that incorrectly factored by grouping was .89 compared to a median score of .94 for the group that correctly answered the problem; 74% of students correctly answered this problem. The effect size implies 1.4% of the variability in ranks was accounted for by the independent variable (i.e., groups incorrect or correct).

Only one technique was taught to solve this type of problem, and it required multiplication-fact skills to solve. Students had to multiply the leading coefficient (in this case, 4) by the constant coefficient (7) to get a product of 28. Then students had to find factors of 28 whose sum was – 11 (the coefficient of x). The factors were – 4 and – 7 because their product was 28 and their sum was – 11. Students would use the factors in the factor-by-grouping algorithm to re-write the initial problem as 4-terms instead of three. At that point, the algorithm required factoring out the greatest common multiple (GCM) from the first two terms, then again from the second two terms, and finishing the algorithmic steps. Finding factors, as well as factoring out the GCMs, requires multiplication-fact skills not easily by-passed with scientific calculators.

Problem 4: Simplify a Rational Expression

Problem 4 contained a rational expression that needed to be simplified. Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups (p = .026). The median score for the group that incorrectly factored by grouping was .85 compared to a median score of .95 for the group that correctly answered the problem; 85% of students correctly answered this problem. The effect size implies 2.6% of the variability in ranks was accounted for by the independent variable (i.e., groups incorrect or correct).

Only one technique was taught to simplify rational expressions. Multiplication-fact skills were required to solve. The process required factoring out the GCM from both the numerator and denominator, then reducing any common factors to achieve a simplified result. In the example given, the GCM of the numerator was 4 because it was a factor of $4x$ and –32. Once 4 was factored out, the remaining binomial was $x - 8$. In the denominator, x was the GCM, leaving $x - 8$ as the remaining binomial. Since the numerator and denominator contained the same factor ($x - 8$), it could be cancelled because any factor divided by itself equals 1. The simplified solution to this problem was the fraction $4/x$. As explained in problem 3, finding a GCM requires multiplication-fact skills not easily circumvented with scientific calculators.

Problem 5: Simplify a Radical Expression

The instructions for problem 5 were to add or subtract add radical expressions. Statistically significant differences existed between median multiplication-fact automaticity scores of the two groups (p = .001). The median score for the group that incorrectly factored by grouping was .89 compared to a median score of .95 for the group that correctly answered the problem; only 67% of students correctly answered this problem. The effect size implies 3% of the variability in ranks was accounted for by the independent variable (i.e., group incorrect or correct). No other technique was taught for this problem.

Students were taught one method and calculators capable of simplifying radicals were not permitted. Thus, multiplication-fact skills were necessary. Students were taught to only add or subtract like terms. However, to discern whether like terms existed in this problem students had to first simplify each radical. For the first term, 125 was the radicand and could be factored into 25 and 5. These two factors were chosen because 25 is a perfect square and could be removed from the radical. The square root (sqrt) of 25, which is 5, could be multiplied together with the 5 that was in front of the radical sign. The product became 25, on the outside, and the simplified term was 25 sqrt 5. Similarly, the radicand of the second term, 28, could be factored into 4 and 7. These factors were chosen because 4 is a perfect square whose square root is 2, leaving only 7 inside the radical. The 2 from the radical could be multiplied with the 2 outside of the radical to result in 4 sqrt 7. The third term could be simplified to 21 sqrt 5. The first and last terms both resulted in having 5 as the radicands, meaning they were like terms and could be combined. The final answer was 4 sqrt 5 – 4 sqrt 7.

Comments

Bloom's taxonomy has been widely accepted as a mechanism to regulate levels of complexity and concreteness (Krathwohl, 2002). Educators frequently have used it as a decision-making tool to improve curricular planning and instructional delivery (Fan & Bokhove, 2014; Jorgensen, 2010; Lee & Huh, 2014; Roegner, 2013; Woodward, 2004). Mathematical concepts range in complexity from basic fact knowledge to multi-layered, abstract problems that challenge the most gifted of mathematicians. Classifying the mathematical continuum of facts, algorithms, and problem-solving within a specific learning objective can reduce problematic concepts by illuminating prerequisite categories which may have been overlooked, or underemphasized (Fan & Bokhove, 2014; Krathwohl, 2002; Roegner, 2013).

This research contributes to the literature in two ways: its consistency with the theoretical framework of Bloom's taxonomy, which suggests mastery at each category is dependent upon mastery in the preceding category (Krathwohl, 2002), and by reducing the gap addressing multiplication-fact automaticity and its influence on student success in developmental mathematics. This study was limited to a volunteer group (n = 365) of Intermediate Algebra students 18 years of age and older from a small university in the southwest United States during the Fall 2017 semester who were not enrolled in sections taught by the researcher. Thus, the results are not generalizable. The sample only included students who completed all five-unit tests, consequently eliminating many of the academically weaker students who stopped attending the course mid-term. Additional limitations included the researcher's inability to discern which technique students used to answer the first two problems, each of which formally included two instructional means for solving.

It is meaningful to consider the practical significance of the ranges between incorrect and correct categories for each of problems 3 (.89 versus .94), 4 (.85 versus .95), and 5 (.89 versus .95). The data support the notion of including automaticity-level multiplication-fact skill development within the Intermediate Algebra class, or as a prerequisite requirement, targeting a multiplication-test score in the low .90's. Future research is recommended to explore the extent to which developmental mathematics courses employ factoring concepts and skills, as well as the dependency factoring has on multiplication-fact automaticity. A meta-analysis of multiplication-fact requirements for developmental mathematics courses across the nation could initiate a conversation of generalizability.

Study 3

The purpose of the third study was to explore, through personal interviews, the lived experiences of students who withdrew from a technology-based, Intermediate Algebra course at a small, southwestern university. Interviews were transcribed verbatim resulting in 123 significant statements. Six themes emerged: Student goals, false course-expectations, the decision to withdraw, mathematics experiences, strategies for success, and mathematics self-efficacy.

The Essence of the Experience

The culmination of these lived stories portrayed the essence of the experience of having withdrawn from Intermediate Algebra at this institution. These students had an earnest desire to successfully complete the course in pursuit of their personal, professional, and educational goals only to be met with a complete misunderstanding of the course demands and expectations. For most of them, hard work was met with "confusion," "frustration," and a sense of "isolation," not knowing how to access help. For those with low mathematic self-efficacy, these feelings were all too familiar as they compared them to memories of pain and hopelessness, save a fleeting moment or two of clarity or relevance, or perhaps a teacher who was able to bring joy to a student's life amid mathematical goings-on. The choice of withdrawing was no choice at all as many looked for alternatives. Alas, the decisions were made with regret for some, others "embarrassment," and defeat. At the dawn of their new plan, all of them shifted their mindsets to prepare for next time. They would "reduce their loads," take care of the medical issue, "buy the textbook," "go to the tutoring lab," "switch teachers," choose a different instructional format, "ask questions," and most of all, not be surprised!

Of particular interest to me as a mathematics teacher were the responses from Theme 6 (Mathematics Self-Efficacy), which described students' perceptions of their multiplication-facts and calculator usage. Their answers revealed a relationship between multiplication-fact skills, calculator dependency, and self-efficacy, as understood through their described mathematical experiences in Theme 4 (Mathematics Experiences). Because self-efficacy has been identified as one of the best predictors of student success, a discussion of self-efficacy is warranted.

Across multiple studies, mathematics self-efficacy tends to be the best predictor among various factors known to predict student success (see Zientek & Thompson, 2010). Self-efficacy has been defined as "the beliefs students hold about their academic capabilities" (Usher & Pajares, 2009, p. 89). Mastery experiences and physiological states are two of four sources of self-efficacy (Bandura, 1997; Usher & Pajares, 2009). Research suggests mastery experiences are the most powerful source of self-efficacy (Usher & Pajares, 2006; Zientek, Fong, & Phelps, 2017) and are acquired through prior successes and failures. In mathematics, physiological states often refer to mathematics anxiety, which has tended to be exhibited at higher levels in developmental mathematics students than the general population (see Zientek, Yetkiner, & Thompson, 2010). Vicarious experiences and social persuasions are the other two sources of self-efficacy

(Bandura, 1997) and have been identified as predictors of developmental mathematics students' skills (Zientek et al., 2017).

The specific calculator functions upon which the students with low MFA felt a dependency, were fractions, division, radicands, signs, and finding factors. Excluding operations with signs, these mathematical computations are heavily dependent on multiplication facts. Further, research has indicated that MFA is a predictor of students' abilities to perform fraction procedures (Hansen et al., 2015), and fraction procedures have been advocated as a predictor of algebra abilities (U. S. Department of Education, 2013; Zientek, Younes, Nimon, Mittag, & Taylor, 2013).

As a final observation, Theme 5 (Strategies for Success), which spoke of implementing change to increase chances of success upon retaking the course, was laden with non-mathematical skills-building strategies. As thorough as the participants were in describing what they learned from their mistakes in the first attempt, it was surprising to discover the void in their abilities to identify specific computational skills that, upon practicing, could contribute to their follow-on success. The lack of preparedness, as it relates to basic skills, coincides with the research of both Barbatis (2010) and Bambara, et. al (2009).

Comments

Symbolic interactionism provides a lens that enables the researcher to consider several actors in the model that may or may not contribute to the students' perceptions and ultimate decisions to withdraw. Such actors included the teachers, time, technology, the physical classroom spaces, students' individual histories of mathematics and learning, cultural predispositions, relevance, workloads, a personal account of MFA and calculator skills, health, and familial and professional responsibilities. Through symbolic interactionism, the researcher can explore how these actors, or variables, prompt action, differ among individuals, and inform change. Symbolic interactionism frequently has been used in the literature to gain understanding of human perception in social engagements and is based on three main tenets: meaning causes action, meaning can be different, and meaning can change.

Understanding the experiences of the participants in this study might prompt change in student-teacher engagement, marketing of resources, understanding the implications of student self-efficacy, personalizing basic-skill improvement strategies, and fostering an appreciation for overwhelming demands inherent of some developmental mathematics courses. While phenomenological studies are not meant to be generalizable, the student voices resonated with current literature highlighting these concerns (Bambara, et al. 2009; Barbatis, 2010; Cordes, 2014; Koch, et al. 2012; Minnick, 2008; Perry, et al. 2008).

This group of students was limited to those who previously had withdrawn but chose to retake the course. A qualitative study illuminating the experiences of students who chose not to return would be enlightening and an interesting contrast to the current study. Additionally, understanding student perspectives of self-efficacy in relation to basic skills and mindset would inform preparation and prerequisite practices.

Final Thoughts

Conducting three separate studies adhering to a single theme provided a richness that could otherwise have been lost. The synthesis of multiplication-fact automaticity regarding Intermediate Algebra students at the hosting institution revealed four primary points. First, students with high MFA had greater success on three tests, end-of-course grade, and three specific test problems, as well as exhibited a higher level of mathematical self-efficacy. Second, a noteworthy relationship emerged in all three studies between MFA and factoring. Third, the percentage for a reasonable cutoff score delineating high and low MFA seemed to be in the low 90's. Fourth, for the aforementioned variables, the availability and use of scientific calculators was insufficient in creating equity.

These findings will inform educational practices at the hosting institution regarding developmental mathematics. Hopefully, while the studies are not generalizable, the results promote the need for a new conversation among educators and administrators investigating the importance of MFA at the primary, secondary, and post-secondary levels. Current literature is lacking evidence to provide sound guidelines about MFA instruction and relevance, rendering stakeholders unequipped to determine best-practices. This study adds to the literature and hopefully evokes similar interest for future studies.

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APPENDIX A

Demographic Questionnaire

1. Did you take Math 0900 at this institution?
2. How many times have you enrolled in Intermediate Algebra at this institution?
3. Have you ever previously withdrawn from this course?
4. If you answered yes to the previous question, would you be willing to participate in an interview about your decision to withdraw?

APPENDIX B

Consent Form

Sam Houston State University

Consent for Participation in Research

Multiplication Facts and the Intermediate Algebra Student

Why am I being asked?

You are being asked to be a participant in a research study about multiplication facts in Intermediate Algebra conducted by Michele Poast, Department of Developmental Education, College of Education at Sam Houston State University and Dixie State University. I am conducting this research under the direction of Dr. Linda Zientek. You have been asked to participate in the research because you are an Intermediate Algebra student at Dixie State University and may be eligible to participate. We ask that you read this form and ask any questions you may have before agreeing to be in the research.

Your participation in this research is voluntary. Your decision whether or not to participate will not affect your current or future relations with Sam Houston State University or Dixie State University. If you decide to participate, you are free to withdraw at any time without affecting that relationship.

Why is this research being done?

This research is a dissertation study designed to gain a deeper understanding of the role that multiplication facts play in Intermediate Algebra success and retention. Data will be gathered from the Institutional Research Department, along with the use of questionnaires, assessments, chapter exams, and interviews. All data, except for the interviews, will be collected from prior records or through MyMathLab at the eLAB testing center. There are no known risks or benefits to the participants. Benefits to the education field include knowledge gained about Intermediate Algebra students' multiplication fact fluency skills and the role these skills have on algebra success. The knowledge can possibly inform educators on course curriculum design.

What is the purpose of this research?

The purpose of this research is to gain a deeper understanding of the value multiplication fact automaticity has on student performance in Intermediate Algebra.

What procedures are involved?

If you agree to be in this research, we would ask you to do the following things:

- Demographic Survey: You will be asked to complete an online background survey to give the researcher a profile of the participants in the study (5-min outside of class on MyMathLab).
- Math Self-Concept Survey: You will be asked to complete a 10-item instrument that measures your beliefs about your mathematics abilities and mathematics in general (5-min outside of class on MyMathLab).
- Pre-Multiplication Fact Skills Assessment: You will be asked to complete an online assessment of pre-multiplication fact knowledge (3 problems within a 5-minute time limit) to the best of your ability (on MyMathLab in the eLAB testing center). ***You will be awarded extra credit in the amount of 0.25% of your overall grade in the Intermediate Algebra course.***
- Multiplication Fact Assessment: You will be asked to complete an online assessment of multiplication facts to the best of your ability (on MyMathLab in the eLAB testing center within a maximum of 5-minutes). ***You will be awarded extra credit in the amount of 0.25% of your overall grade in the Intermediate Algebra course.***
- Interview: You may be contacted to participate in a 30-minute interview. Participation is voluntary and will not affect your participation in the prior survey/assessments. The interviews will take place in the researcher's office on the Dixie State University campus. At the time of the interview, a set of questions pertaining to your experiences with mathematics, and your decision to withdraw or complete the course will be asked. The interview will be audio-recorded for transcription/research purposes and the audio-recordings will be destroyed upon transcription of the interviews. Your identity will not be disclosed to non-research personnel. ***Extra credit will be awarded in the amount of 0.5% of your overall grade in the Intermediate Algebra course.***

None of the above information will be available to your instructor at any time and, other than interview participants, your identity will not be available to the researcher. All research data will be destroyed within three years of completion of the study. Approximately 2000 participants may be involved in this research at Dixie State University.

What are the potential risks and discomforts?

This research may be considered an inconvenience to participants as it will consume some time otherwise dedicated to the course activities. There are no significant physical or psychological risks and participants can stop at any time.

Are there benefits to taking part in the research?

The outcome of this study will enable the researchers to collect knowledge pertaining to developmental math success and retention. Such knowledge will increase understanding of the experiences of developmental math students. Succinctly put, your involvement in this study may or may not increase your awareness of personal decisions regarding developmental math courses. (No monetary compensation will be given for this study).

What other options are there?

Participants may choose to complete the assessments in a different location outside of the classroom or not participate at all.

What about privacy and confidentiality?

The only people who will know that you are a research participant are professional data collectors from the Institutional Research Departments, and the Mathematics Department Data Analyst. In addition, the identities of the 12 participants who volunteer to be interviewed will be known to the researcher/interviewer. No information about you, or provided by you during the research will be disclosed to others without your written permission, except:

- if necessary to protect your rights or welfare (for example, if you are injured and need emergency care or when the SHSU Protection of Human Subjects monitors the research or consent process); or
- if required by law.

When the results of the research are published or discussed in conferences, no information will be included that would reveal your identity. Audiotape recordings will be destroyed upon transcriptions of the interviews. In the event that transcriptions will be used for educational purposes, your identity will be protected or disguised. Any information that is obtained in connection with this study and that can be identified with you will remain confidential and will be disclosed only with your permission or as required by law.

Pertinent information about you will be stored safely in the researcher's locked file cabinet and locked office. All information gathered from this study will be analyzed holistically. Participant names will not be used. Audio tapes will be heard only for research purposes by the investigator and her associates, and destroyed upon transcription. If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

Interviewees maintain the right to review/edit the audio-recordings and transcriptions. The transcriptions will be password protected and/or stored in a locked container during use, and destroyed within three years of completion of the research.

What if I am injured as a result of my participation?

In the event of injury related to this research study, you should contact your physician or the University Health Center. However, you or your third party payer, if any, will be responsible for payment of this treatment. There is no compensation and/or payment for medical treatment from Sam Houston State University or Dixie State University for any injury you have from participating in this research, except as may by required of the University by law. If you feel you have been injured, you may contact the researcher, *Michele Poast* at *(435) 652-7918.*

What are the costs for participating in this research?
There are no costs for participating in this research.

Will I be reimbursed for any of my expenses or paid for my participation in this research?
There is no monetary compensation for participation in this study. However, some extra credit will be awarded as follows.

- Pre-Multiplication Fact Skills Assessment: You will be awarded extra credit in the amount of 0.25% of your overall grade in the Intermediate Algebra course.
- Multiplication Fact Assessment: You will be awarded extra credit in the amount of 0.25% of your overall grade in the Intermediate Algebra course.
- Interview: Extra credit will be awarded in the amount of 0.5% of your overall grade in the Intermediate Algebra course.

Can I withdraw or be removed from the study?

You can choose whether to be in this study or not. If you volunteer to be in this study, you may withdraw at any time without consequences of any kind. You may also refuse to answer any questions you do not want to answer and still remain in the study. The investigator may withdraw you from this research if circumstances arise which warrant doing so.

Who should I contact if I have questions?

The researcher conducting this study is Michele Poast. You may ask any questions you have now. If you have questions later, you may contact the researcher at: (435) 652-7918, or the researcher's advisor, Dr. Linda Zientek, at (936) 294-4874.

What are my rights as a research subject?

If you feel you have not been treated according to the descriptions in this form, or you have any questions about your rights as a research participant, you may call the Office of Research and Sponsored Programs – Sharla Miles at 936-294-4875 or e-mail ORSP at sharla_miles@shsu.edu.

You may choose not to participate or to stop your participation in this research at any time. Your decision whether or not to participate will not affect your current or future relations with Sam Houston State or Dixie State Universities.

If you are a student, this will not affect your class standing or grades at SHSU or DSU. The investigator may also end your participation in the research. If this happens, your class standing or grades will not be affected.
You will not be offered or receive any special consideration if you participate in this research.

<u>Agreement to Participate</u>

I have read (*or someone has read to me)* the above information. I have been given an opportunity to ask questions and my questions have been answered to my satisfaction. I agree to participate in this research.

Consent: I have read and understand the above information, and I willingly consent to participate in this study. I understand that if I should have any questions about my rights as a research subject, I can contact Michele Poast at (435) 652- 7918 or by email at poast@dixie.edu. I have received a copy of this consent form.

____I agree to participate ______I do not agree to participate

APPENDIX C

Interview Protocol

Time of interview:

Date:

Place: SNOW 115, DSU

Interviewer: Michele Poast

Interviewee:

Position of interviewee: Student at Dixie State University

This is a phenomenological study about students who chose to withdraw from an Intermediate Algebra course at a small, southwestern university.

I. Establish Rapport

   a. Tell me a little about yourself.

   b. Why did you choose to go to college?

II. Descriptive Questions

   a. Grand tour

      i. Tell me about a typical day in your dropped Intermediate Algebra class.

      ii. How would you describe your relationship with the instructor of the dropped course?

      iii. Tell me about your decision to withdraw from Intermediate Algebra.

      iv. Describe the study skills you used in the dropped course.

v. Tell me about your experiences memorizing multiplication facts. How did you learn them? What strategies did you find most helpful?

vi. Do you always use a calculator when solving a mathematics problem?

a. If so, is it because you do not know your multiplication facts or for other reasons? What other reasons?

vii. What might have helped you to remain in the Intermediate Algebra class?

b. Mini tour – tell me more about that

c. Example – what would be an example of that

d. Experience

i. Tell me about a positive mathematics experience.

ii. Tell me about a negative mathematics experience.

e. Native language

i. What kind of language do people use to describe developmental math classes, ie Intermediate Algebra?

III. Structural Questions – follow-up, probing, specifying, interpreting

Questions in order:

1. Tell me a little about yourself.
2. Why did you choose to go to college?
3. Tell me about a positive mathematics experience.
4. Tell me about a negative mathematics experience.

5. What kind of language do people use to describe developmental math classes, i.e. Intermediate Algebra?
6. Tell me about your decision to withdraw from the Intermediate Algebra course.
7. Tell me about a typical day in your dropped Intermediate Algebra class.
8. How would you describe your relationship with the instructor of the dropped class?
9. Describe the study skills you used in the dropped course.
10. Tell me about your experiences memorizing multiplication facts. How did you learn them? What strategies did you find most helpful?
11. Do you always use a calculator when solving a mathematics problem?

    a. If so, is it because you do not know your multiplication facts or for other reasons? What other reasons?
12. What might have helped you to remain in the Intermediate Algebra class?

APPENDIX D

Version Date: 03/31/2011

Sample text for an Institution with a Federalwide Assurance (FWA) to rely on the IRB/IEC of another institution (institutions may use this sample as a guide to develop their own agreement).

Institutional Review Board (IRB) Authorization Agreement

Name of Institution or Organization Providing IRB Review (Institution/Organization A):
Sam Houston State University

IRB Registration #: 0002245 Federalwide Assurance (FWA) #, if any: 00002405

Name of Institution Relying on the Designated IRB (Institution B):
Dixie State University

FWA #: 00023200

The Officials signing below agree that Dixie State University may rely on the designated IRB for review and continuing oversight of its human subjects research described below: (*check one*)

( ) This agreement applies to all human subjects research covered by Institution B's FWA.

(X) This agreement is limited to the following specific protocol(s):

Name of Research Project: Multiplication Facts and the Intermediate Algebra Student
Name of Principal Investigator: Michele Poast
Sponsor or Funding Agency: __________ Award Number, if any: __________

( ) Other (*describe*): __________

The review performed by the designated IRB will meet the human subject protection requirements of Institution B's OHRP-approved FWA. The IRB at Institution/Organization A will follow written procedures for reporting its findings and actions to appropriate officials at Institution B. Relevant minutes of IRB meetings will be made available to Institution B upon request. Institution B remains responsible for ensuring compliance with the IRB's determinations and with the Terms of its OHRP-approved FWA. This document must be kept on file by both parties and provided to OHRP upon request.

Signature of Signatory Official (Institution/Organization A):
__________ Date: 11 July 2017

Print Full Name: Chad Hargrave Institutional Title: Associate Vice President for Research

NOTE: The IRB of Institution A may need to be designated on the OHRP-approved FWA for Institution B.

Signature of Signatory Official (Institution B):
__________ Date: 7/6/17

Print Full Name: Michael LaCourse Institutional Title: Provost and Vice President of Academic Affairs

VITA

Michele E. Poast
Dixie State University
Mathematics

Education

EDD, Sam Houston State University, 2018.
Major: Developmental Education Administration
Supporting Areas of Emphasis: Developmental Mathematics
Dissertation Title: Multiplication Facts and the Intermediate Algebra Student

MS, Fayetteville State University, 1999.
Major: Mathematics

BA, Hawaii Pacific University, 1995.
Major: Mathematics

Academic, Government, Military and Professional Positions

Academic - Post-Secondary

Chair, Student Conduct Committee. (August 21, 2013 - August 24, 2017).

Professional

Director of Operations/Executive Director, The Erin Kimball Memorial Foundation. (October 2009 - June 2012).

Licensures and Certifications

Child Abuse: Mandatory Reporting, Safe Colleges. (December 13, 2016 - Present).

Diversity Awareness: Staff to Staff, Safe Colleges. (December 12, 2016 - Present).

Sexual Harassment: Staff to Staff, Safe Colleges. (December 12, 2016 - Present).

Title IX and Sexual Misconduct, Safe Colleges. (December 12, 2016 - Present).

Certificate of Completion, Safe Colleges. (December 14, 2015 - Present).

Certificate of Completion, Safe Colleges. (December 10, 2015 - Present).

Certificate of Completion, Safe Colleges. (December 9, 2015 - Present).

Certificate of Completion, Safe Colleges. (October 8, 2015 - Present).

Certificate of Completion, Safe Colleges. (October 7, 2015 - Present).

Certificate of Completion, Safe Colleges. (October 6, 2015 - Present).

Collaborative Institutional Training Initiative (CITI), University of Miami. (September 17, 2014 - Present).

Online Course Delivery Certification, DSU. (December 17, 2013 - Present).

PC Construction/Computer Repair Recognition of Training, DXATC. (October 4, 2012 - Present).

Professional Memberships

National Association of Developmental Education. (February 25, 2015 - Present).

Development Activities Attended

Doctoral Degree, "Developmental Education Administration," Sam Houston State University, Huntsville, Texas, USA. (May 27, 2014 - Present).

Continuing Education Program, "Endorsement to Teach and Create Online Courses," Dixie State University, St. George, Utah, USA. (October 22, 2012 - December 17, 2013).

Continuing Education Program, "PC Construction/Computer Repair," DXATC, St. George, Utah, USA. (September 4, 2012 - October 4, 2012).

TEACHING

Teaching Experience

Dixie State University
MATH, 3 courses.
MATH 0900, Transitional Math I, 5 courses.
MATH 0980, Transitional Mathematics IIB, 2 courses.
MATH 1000, Transitional Mathematics II, 21 courses.
MATH 1010, Intermediate Algebra, 12 courses.
MATH 1040, 1 course.
MATH 1050, PreCalculus, 33 courses.
MATH 1090, MATH, 1 course.

MATH 1100, Business Calculus, 2 courses.
MATH 1210, MATH, 2 courses.
MATH 1220, MATH, 2 courses.
MATH 2010, Math for Elementary Teachers I, 7 courses.
MATH 2020, Math for Elementary Teachers II, 5 courses.
MATH 2210, MATH, 1 course.
MATH 2270, MATH, 1 course.
MATH 920, Basic Math/Pre-Algebra, 1 course.
MATH 990, Elementary Algebra, 1 course.
PBC 0700, Math for Placement A, 2 courses.
PBC 0700R, Math for Placement A/ONLINE, 2 courses.
PBC 0800, Math for Placement I, 4 courses.
PBC 0800R, Math for Placement I/ONLINE, 2 courses.
PBC 1000, Math for Placement II/ONLINE, 20 courses.
PBC 1000R, Math for Placement II/ONLINE, 2 courses.
PBC 700, Math for Placement A/ONLINE, 12 courses.
PBC 800, Math for Placement I/ONLINE, 20 courses.

RESEARCH

Presentations Given

Poast, M. E., NADE - 2016, "Do Multiplication Facts Matter?" National Association of Developmental Education, Anaheim, CA. (March 18, 2016).

Poast, M. E., SERA – 2018, “Multiplication Facts and Intermediate Algebra Success: An Exploration.” Southwest Educational Research Association, New Orleans, LA. (February 14, 2018).

Research Currently in Progress

"Multiplication Facts and the Intermediate Algebra Student" (On-Going). The purpose of this research is to understand the role multiplication fact automaticity has on student success and retention in Intermediate Algebra.

Zientek, L., Jones, D., Swarthout, M., Ozel, S., Poast, M., & Nickerson, S., “Predictors of Solving Equations in One or Two Variables.”
The purpose of this study is to quantify degrees of relationships between various mathematics skills and students’ abilities to solve arithmetic algebra equations and equations with two variables.

SERVICE

Department Service

Committee Member, Transition Math. (August 1, 2013 - July 31, 2014).

Faculty Mentor, Evaluation of Concurrent Enrollment Instructor. (January 8, 2014 - May 3, 2014).

Committee Member, Hiring Committee. (May 1, 2013 - May 31, 2013).

Committee Chair, Developmental Math Majors. (September 6, 2011 - September 30, 2011).

University Service

Committee Member, IIAC. (August 20, 2014 - Present).

Task Force Member, Safe Zone. (February 10, 2014 - Present).

Committee Member, ESL. (August 20, 2013 - Present).

Faculty Mentor, Non-Committee Service to University. (August 21, 2012 - Present).

Chairperson, Student Conduct Committee. (August 21, 2012 - Present).

Special Institutional Assignment, Health and Wellness Surveys. (February 9, 2015 - February 11, 2015).

Committee Member, Hiring Committee. (February 1, 2014 - May 31, 2014).

Attendee, Meeting, Women's Mentoring. (August 15, 2013).

Committee Member, Math Majors. (August 2009 - December 2012).

Grant Proposal Reviewer, Internal, Title III Grant Committee. (April 2010 - April 2011).

Professional Service

Member, National Association for Developmental Education, Kinnelon, NJ. (February 25, 2015 - Present).

Public Service

Workshop Instructor, ACT Prep Course, St. George, Utah. (September 15, 2014 - Present).

Attendee, Meeting, DocUtah, St. George, Utah. (September 4, 2012 - Present).

Attendee, Meeting, Equality Utah, St. George, Utah. (May 9, 2016).

Program Organizer, The Heart Walk Foundation, St. George, Utah. (March 1, 2014 - April 2, 2014).