Math 2440 – Spring 2017 **WS #6 – Multivariate Functions** Name: ______________

1. Describe the graph of each function below. Be as specific as possible.

   (a) $f(x,y) = x^2 + y^2$

   (b) $g(x,y) = 5x^2 + y^2$

   (c) $h(x,y) = \sqrt{16 - x^2 + y^2}$

   (d) $s(x,y) = x^2 - y^2$

We say $\lim\limits_{(x,y)\to(a,b)} [f(x,y)] = L$ if the function values of f get closer to L, *no matter how* (x,y) *approaches the point* (a,b).

2. Let $f(x,y) = \dfrac{3x-2y}{x^2+y}$. Let's try to see if the limit of $f(x,y)$ exists as $(x,y) \to (0,0)$.

   (a) If we approach the origin along the line $x = 0$, then

$$\lim_{(0,y)\to(0,0)} \left[\frac{3x-2y}{x^2+y}\right] = \lim_{y\to 0} \left[\frac{-2y}{y}\right] =$$

   (b) If we approach the origin along the line $y = 0$, then

$$\lim_{(x,0)\to(0,0)} \left[\frac{3x-2y}{x^2+y}\right] =$$

   (c) What can you conclude about $\lim\limits_{(x,y)\to(0,0)} \left[\dfrac{3x-2y}{x^2+y}\right]$??

3. Let $g(x,y) = \dfrac{x^2y^2}{x^4+3y^4}$. Let's try to see if the limit of $f(x,y)$ exists as $(x,y) \to (0,0)$.

(a) If we approach the origin along the line $x = 0$, then

$$\lim_{(0,y)\to(0,0)} [g(0,y)] =$$

(b) If we approach the origin along the line $y = 0$, then

$$\lim_{(x,0)\to(0,0)} [g(x,0)] =$$

(c) If we approach the origin along the line $x = y$, then

$$\lim_{(x,y)\to(0,0),\, x=y} [g(x,x)] =$$

(d) What can you conclude about $\displaystyle\lim_{(x,y)\to(0,0)} \left[\frac{x^2y^2}{x^4+3y^4}\right]$??

4. What about $h(x,y) = \dfrac{y}{x+y-1}$ as $(x,y) \to (1,0)$? [Hint: let (x,y) approach the point along vertical and horizontal lines.]