Math 2440 – Spring 2017 **WS #8 – Tangent planes** Name: ________________

1. Let's find the equation of the plane that is tangent to the sphere $x^2 + y^2 + z^2 = 14$ at the point $P = (1, 3, -2)$.

   (a.) Find a function $F(x, y, z)$ and a constant c so that the equation above is a level surface for F.

   (b.) Compute the gradient ∇F of F.

   (c.) Evaluate the gradient of F at the point P, denoted $\nabla F \Big|_P$.

   (d.) What do you know about this vector?

   (e.) Use this to find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 14$ at the point $P = (1, 3, -2)$.

2. Find the equation of the tangent plane to the surface $4x^2 + 9y^2 - z^2 = 16$ at the point $(2, 1, 3)$.

3. The graph of the function $f(x, y) = 4x^2 - 3y^2$ is a hyperbolic paraboloid, or a "saddle". Find the equation of the plane tangent to this saddle at the point $(3, 4, -12)$.

   [Hint: the graph is the set of points that satisfy $z = f(x, y)$.]

4. The spheres below intersect at a circle $\mathcal{C}$. The point $P = (1, 1, 1)$ is on this circle of intersection.

$$x^2 + y^2 + z^2 = 3 \qquad\qquad (x-2)^2 + (y-2)^2 + z^2 = 3$$

   If these equations are thought of as level surfaces of functions $F(x, y, z)$ and $G(x, y, z)$, what can be said about the vector $\nabla F \times \nabla G$ at P? Find the equation of the tangent line to $\mathcal{C}$ at P.