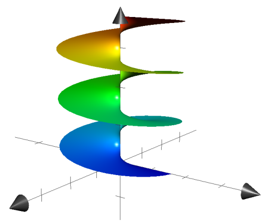
Math 2440 – Spring 2017 **WS #12 – Parametric Surfaces** Name: ______________

A **parametric surface** is similar to a parametric curve $\overrightarrow{r}(t)$, with the addition of a second parameter:

$$\overrightarrow{r}(u,v) = \langle x(u,v),\, y(u,v),\, z(u,v)\rangle, \quad a \le u \le b,\ c \le v \le d$$

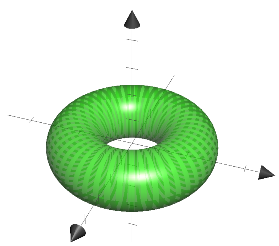
For example, a particular *helicoid* has parameterization

$$\overrightarrow{r}(u,v) = \langle u\cos v,\, u\sin v,\, v\rangle, \quad 0 \le u \le 5,\ 0 \le v \le 6\pi$$



A particular *torus* has parameterization

$$\overrightarrow{r}(u,v) = \langle (2+\cos u)\cos v,\, (2+\cos u)\sin v,\, \sin u\rangle, \quad 0 \le u \le 2\pi,\ 0 \le v \le 2\pi$$



1. On the helicoid defined above: $\overrightarrow{r}(u,v) = \langle u\cos v,\, u\sin v,\, v\rangle, \quad 0 \le u \le 5,\ 0 \le v \le 6\pi$

   (a) Compute the vector $\overrightarrow{r_u} = \frac{\partial \overrightarrow{r}}{\partial u}$ of partial derivatives with respect to the parameter u.

   (b) Compute $\overrightarrow{r_v} = \frac{\partial \overrightarrow{r}}{\partial v}$ as well.

   (c) What can be said about the direction of these two vectors (in relation to the surface itself)?

   (d) What can be said about the vector $\overrightarrow{r_u} \times \overrightarrow{r_v}$?

2. Compute the equation of the plane tangent to the helicoid above at the point corresponding to $(u, v) = (3, \pi)$.

3. Compute the surface area of the helicoid on the domain $0 \leq u \leq 5,\ 0 \leq v \leq 6\pi$.

4. Compute the surface area of the torus above on its domain $0 \leq u \leq 2\pi,\ 0 \leq v \leq 2\pi$