Math 2440 – Spring 2017 **WS #7 – Chain Rule and tangent lines** Name: ________________

1. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $z = f(x, y) = x^2 \sin y$.

   (a.) Compute $f(0, 0)$, $f(\pi, \pi)$, $f(2, \frac{\pi}{2})$, and $f(3, \frac{\pi}{2})$.

   (b.) Suppose y is fixed to be equal to $\frac{\pi}{2}$. What is $z = f(x, \frac{\pi}{2})$? What shape is this curve $z = f(x, \frac{\pi}{2})$?

   (c.) Suppose y is fixed to be a constant y_0. What is $z = f(x, y_0)$? What shape is the curve $z = f(x, y_0)$?

   (d.) Suppose x is fixed to be a constant x_0. What is $z = f(x_0, y)$? What shape is the curve $z = f(x_0, y)$?

   (e.) Back to the case of $y = \frac{\pi}{2}$. Let's find the tangent line at $x = 3$. What is its slope?

   (f.) In what plane does this tangent line live? Write the equation of this line in this plane.

   (g.) Write a parameterization $\overrightarrow{r_x}(t)$ for this line in space.

   (h.) Now, what if we fix $x = 3$? Find a parameterization $\overrightarrow{r_y}(t)$ for the tangent line at $(x, y) = (3, \frac{\pi}{2})$.

   (i.) BONUS: Find the equation of the plane tangent to the graph of f at $(x, y) = (3, \frac{\pi}{2})$.

2. Consider the function $g(x, y) = 5x^2 - 3y^2$. Find parameterizations of two lines tangent to the graph of g at the point $(2, 1, 17)$.

   [BONUS: Find the equation of the plane tangent to the graph at that point.]