Math 2440 – Spring 2017 **WS #14 – Conservative Vector Fields** Name: ______________

Yesterday we defined a **vector field** as a function:

$$F : \mathbb{R}^2 \to \mathbb{R}^2 \qquad \text{or } F : \mathbb{R}^3 \to \mathbb{R}^3$$

We defined a vector field F to be **conservative** if there is a function f so that $F = \nabla f$, and we call f the **potential function** for the conservative vector field F.

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1. Suppose a general vector field $F = \langle M, N, P \rangle$ is conservative. Then there is a function f so that $F = \nabla f$:

$$F = \langle M, N, P \rangle = \langle f_x, f_y, f_z \rangle$$

   (a) What can you say about the partial derivatives M_y and N_x?

   (b) What about P_x?

   (c) Any other interesting associations?

2. We can use this to develop a test to determine whether a vector field is conservative:

   The vector field $F = \langle M, N, P \rangle$ is conservative if and only if three things are true:

   1.
   2.
   3.

3. Use this test to determine if the vector fields below are conservative:

   (a) $F = \langle 8x \sec y, 4x^2 \sec y \tan y, -3z^2 \rangle$

   (b) $F = \langle e^x(z+1), -\cos y, e^x \rangle$

   (c) $F = \langle 4x^3y^2 - ze^x, 2x^4y + \cos z, -e^x - y \sin z \rangle$

4. Is $F = \langle 2xy + 5,\ x^2 - 4z,\ -4y \rangle$ conservative? If so, find its potential function.

5. For the vector fields in #3 that are conservative, what are their potential functions?

6. For a vector field $F = \langle M,\, N,\, P\rangle$ in space, we define the **curl** of F to be the vector field

$$\text{curl}(f) = \langle P_y - N_z,\, M_z - P_x,\, N_x - M_y\rangle$$

or, for short: $\text{curl}(F) = \nabla \times F$.

If $F = \langle 4x^3,\, 3z^2x,\, y^5z\rangle$, then compute $\text{curl}(F)$.

7. If F is a conservative vector field, what can be said about the vector field $\text{curl}(F)$?

8. So the test for conservativity of a vector field in #2 above may be rewritten:

The vector field $F = \langle M,\, N,\, P\rangle$ is conservative if and only if ...

9. Discuss why $\text{curl}(F) = \nabla \times F$ makes sense with the definition above.

10. For a vector field $F = \langle M, N, P \rangle$ in space, we define the **divergence** of F to be the function

$$\mathrm{div}(f) = M_x + N_y + P_z$$

or, for short: $\mathrm{div}(F) = \nabla \cdot F$.

If $F = \langle 4x^3, 3z^2x, y^5z \rangle$, then compute $\mathrm{div}(F)$.

11. Discuss why $\mathrm{div}(F) = \nabla \cdot F$ makes sense with the definition above.

12. For a vector field $F = \langle M, N, P \rangle$, what can you say about $\mathrm{div}(\mathrm{curl}(F))$?