Math 2440 – Spring 2017 **WS #9 – Extrema** Name: ________________

1. Complete each of the following sentences about the gradient of a function of more than one variable.

   (a.) If f is a function of two variables x and y, then $\nabla f =$

   (b.) If f is a function of three variables x, y and z, then $\nabla f =$

   (c.) The rate of change of f in the direction of a vector $\overrightarrow{v}$ can be found to be $D_{\overrightarrow{v}} f =$

   (d.) If $\mathcal{C}$ is a level curve of $f(x, y)$, then ∇f is a vector ________________ to $\mathcal{C}$.

   (e.) If $\mathcal{S}$ is a level surface of $f(x, y, z)$, then ∇f is a vector ________________ to $\mathcal{S}$.

   (f.) The vector ∇f points in the direction corresponding to ...

2. Suppose f is a function of two variables x and y.

   (a.) What does it mean for (a, b) to be a *local minimum* for f?

   (b.) What does it mean for (a, b) to be a *local maximum* for f?

   (c.) If (a, b) is a local minimum or maximum, what can be said about $f_x = \frac{\partial f}{\partial x}$ at that point?

   (d.) If (a, b) is a local minimum or maximum, what can be said about $f_y = \frac{\partial f}{\partial x}$ at that point?

   (e.) If (a, b) is either a local minimum or maximum, what can be said about ∇f at that point?

   (f.) We define (a, b) to be a *critical point* for f if ...

3. Find all critical points for each of the following functions.

(a.) $f(x, y) = x^2 - y^2 - x - 4y$

(b.) $g(x, y) = 2x^2 + 2xy + y^2 + 2x - 3$

(c.) $h(x, y) = y^3 - 3x^2y - 3y^2 - 3x^2 + 1$

4. For a function $f(x, y)$, define the discriminant d of f as

$$d = (f_{xx})(f_{yy}) - (f_{xy})^2$$

Compute the discriminant for each of these functions:

(a.) $f(x, y) = x^2 - y^2 - x - 4y$

(b.) $g(x, y) = 2x^2 + 2xy + y^2 + 2x - 3$

(c.) $h(x, y) = y^3 - 3x^2y - 3y^2 - 3x^2 + 1$

5. Use the *Second Partials Test* to determine if any of the critical points you found earlier are local maxima or minima for any of the following functions.

   (a.) $f(x, y) = x^2 - y^2 - x - 4y$

   (b.) $g(x, y) = 2x^2 + 2xy + y^2 + 2x - 3$

   (c.) $h(x, y) = y^3 - 3x^2y - 3y^2 - 3x^2 + 1$

6. Find all local maxima and minima for the function $f(x, y) = \left(\frac{1}{2} - x^2 + y^2\right) e^{1-x^2-y^2}$