Math 2440 – Spring 2017 **WS #3 – Cross Products** Name: ________________

1. Suppose $\overrightarrow{u} = \langle u_1, u_2, u_3 \rangle$ and $\overrightarrow{v} = \langle v_1, v_2, v_3 \rangle$.

   (a) Compute $\overrightarrow{u} \times \overrightarrow{v}$.

   (b) Compute (and expand) $\|\overrightarrow{u} \times \overrightarrow{v}\|^2$

   (c) Now, compute (and expand) $\|\overrightarrow{u}\|^2 \cdot \|\overrightarrow{v}\|^2 - (\overrightarrow{u} \cdot \overrightarrow{v})^2$

   (d) How do these last two expressions compare?

2. Suppose θ is the angle between $\overrightarrow{u}$ and $\overrightarrow{u}$.

   (a) Use the property of the dot product derived on the last worksheet to rewrite

   $$\|\overrightarrow{u}\|^2 \cdot \|\overrightarrow{v}\|^2 - (\overrightarrow{u} \cdot \overrightarrow{v})^2$$

   (b) Rewrite this using another trigonometry function.

   (c) Simplify as much as possible.

3. We have just proved the following formula:

$$\|\overrightarrow{u} \times \overrightarrow{v}\| =$$

4. Suppose $\overrightarrow{u}$ and $\overrightarrow{v}$ are vectors in $\mathbb{R}^3$ in standard position. Compute the area of the resulting parallelogram.

5. Suppose $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ are vectors in $\mathbb{R}^3$ in standard position. These three vectors form a *parallelipiped*. Compute its volume.