Math 2440 – Spring 2017 **WS #2 – Dot Products** Name: ________________

1. Suppose $\overrightarrow{u}$ and $\overrightarrow{v}$ are any vectors in standard position.

   (a) Determine a way to find a third vector which would form a triangle with $\overrightarrow{u}$ and $\overrightarrow{v}$.

   (b) What are the lengths of the three sides of this triangle?

   (c) If θ is the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$, write an equation involving the lengths of the sides.

2. (a) Use one of the properties of the dot product to rewrite $\|\overrightarrow{u} - \overrightarrow{v}\|^2$ as a product.

   (b) Use the distributive property of dot products to expand this product.

3. (a) We now have two quantities that are equal to the same expression. Set them equal and simplify.

(b) Rewriting this simplified equation, we see that

$$\boxed{\overrightarrow{u} \cdot \overrightarrow{v} = \qquad\qquad\qquad}$$

4. Find the angle between the vectors $\langle 2, 3\rangle$ and $\langle -1, 2\rangle$.

5. Find the angle between the vectors $\langle 0, -1, 2, 3\rangle$ and $\langle -1, 0, 4, 1\rangle$.

6. If the angle between two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ is $\frac{\pi}{2}$, then what can be said about $\overrightarrow{u} \cdot \overrightarrow{v}$?

7. What can be said about $\overrightarrow{u}$ and $\overrightarrow{v}$ if $\overrightarrow{u} \cdot \overrightarrow{v} = 0$?

8. What can be said about $\overrightarrow{u}$ and $\overrightarrow{v}$ if $\overrightarrow{u} \cdot \overrightarrow{v} > 0$?

9. What can be said about $\overrightarrow{u}$ and $\overrightarrow{v}$ if $\overrightarrow{u} \cdot \overrightarrow{v} < 0$?

10. We say $\overrightarrow{u}$ and $\overrightarrow{v}$ are **orthogonal** if ...

11. Write $\langle 2, 3 \rangle$ as a linear combination of the orthogonal vectors $\overrightarrow{i}$ and $\overrightarrow{j}$.

12. Decompose the vector $\overrightarrow{u} = \langle 2, 3 \rangle$ into some part of the vector $\overrightarrow{v} = \langle 4, 1 \rangle$ and another vector orthogonal to $\overrightarrow{v}$.